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Shape Grammars in Transport and Urban Design

Review, Terminology, Assessment, and Application

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Abstract: Shape grammars for urban design have attracted much interest in research and practice. Transport and urban planners increasingly deploy shape grammars especially in models and simulations. However, little is known about the effectiveness of shape grammar in transport networks and urban environments.

The authors posit that grammars require specified and corresponding objectives and application specifications for enhanced implementation. The proposed methodology aims at a future development of a robust and effective language for e.g. sustainable urban development. The proposed methodology for grammar assessment bases on elasticities. Elasticities allow comprehensive comparisons and verification between grammar rules.

First, the paper reviews and highlights the key achievements and applications of shape grammars in cognate fields of science. Terminology sheds light on the definitions of most relevant terms. The consecutive section differentiates methodological approaches in grammar design assessment, and emphasizes a standardized approach for shape grammars developments. The paper concludes with a detailed example for grammar rule assessment, and potential future research.

Keywords: Shape grammars; Urban; Rules; Transport; Network; Design; Assessment; Boulevard.

1 Introduction

The major transport networks and modes coevolved substantially over time. For each mode, urban patterns have been adopted during the last centuries (Jacobs 1993). In the near future, urban agglomerations and systems will grow rapidly around the globe (World Bank 2013), requiring new built environment and transport systems. Planners need to account for economy, social and environmental factors when (re)designing urban patterns and (re)defining urban

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densities. Welfare and productivity highly depend on transport systems due to the degree of economical specialization. However, the optimal urban design for economical, social and environmental needs remains contentious.

An integrated perspective is fostered in recent years for urban planning due to the strong interdependencies of urban systems and components. Travel behavior is linked with activity patterns, and spatially distributed land use types. New modes, and changing living and travel behavior potentially require redesigns of existing urban areas. An example is the ongoing energy-related debate in many countries about future power supply and energy turnaround, and about new transport modes (e.g. Gil and Duarte 2013; Keirstead and Shah 2011). The recent integrative perspective melds these aspects for improved planning.

Research is needed to support urban planning guidelines, even though different transport institutions provide handbooks for network design (AASHTO 2004; FGSV 2008; IHT 1997; VSS 1994). Patterns and rules have been proposed in these handbooks for urban and transport design. These are merely describe the current major transport modes, and often are rule of thumbs and initial assertions. Therefore, it is essential to provide effective rules relying on a fundamental research base. Shape grammars contribute to transport network design, especially if they are established on a solid and quantitative evaluation.

This paper highlights the importance of shape grammars for transport and urban planning and their standardization in design handbooks. Shape grammars define how urban elements are joined with each other to design or redesign urban living environments. We claim that some shape grammars are capable of solving urgent questions in urban design.

The aim of this paper is threefold. (1) to enhance the understanding of shape grammars by a detailed terminology, review examples, and a wider perspective on cognate fields of urban and transport planning. The authors posit that the potential of shape grammars can be exploit in a broader planning context. Established shape grammars in multiple fields enable synergies, and will nurture future interdisciplinary planning applications. (2) to provide a general overview of the current state of shape grammar development. The paper does not claim to provide a complete literature review, but an overview and key achievements in the most relevant fields. (3) to propose a systematic methodology for future shape grammars in transport and urban planning. A systematic methodology enhances cross-disciplinary research, synergies and applications. The methodology consolidates various achievements in the field of shape grammars. An example leads through each stage of the methodology.

2 Terminology

This section briefly defines the most relevant grammar-related expressions. The reader is referred to the wider literature of linguistics, grammars and cognate fields for further details about the relevant terminology.

2.1 Urban Patterns

The meaning of pattern is twofold. Patterns can describe an extracted spatial form which is made of a number of elementary building blocks. Patterns often refer to a particular geometric layout, as a scale plan, featuring absolute position and lengths. A pattern can be used as an archetype for future planning. Example patterns are layouts often used in design handbooks (AASHTO 2004; FGSV 2008; VSS 1994). Lynch (2001) mentioned star, grid, axial, nested, and other kinds of patterns, similar to Marshall (2005). Patterns of different typologies can be recombined in urban planning. Patterns are assessed and compared in science (Estrada *et al.* 2011; Snellen *et al.* 2002; Xie and Levinson 2007).

However, a specific pattern can be the outcome of an applied grammar rule (Alexander *et al.* 1977), which is in contrast to the above definition. In a bottom-up approach, no preconceived pattern exists; urban patterns unfold incrementally (Marshall 2005). So the result is an assembly of urban elements, called pattern. Alexander *et al.* (1977) describes the unfolding process in their seminal books "pattern language".

2.2 Syntax

The syntax encloses the rules to govern distinct elements, like words in linguistics, or elementary building blocks in urban planning. The expression is widely used in computer science to describe the combination of elements to build up a structured source code. Also, architects and urban planners often refer to the syntax, and assemble urban elements according to distinct rules.

The following definition extracts the major components of the earliest definition (Chomsky 1956, 1959). The syntax describes in the form of a finite number of rules how elements e of the same or different type is added to each other. I defines the initial assertion where the algorithm starts. E is the finite set of non-terminal elements e, which are elementary building blocks in urban planning. R is a set of rules r in the form of $\alpha \to \beta$, where $(\alpha, \beta) \in E$. R combines the e to form a certain urban neighborhood, or transport system. R includes rules to stop the algorithm after initialization. The rules describe how given planning states and urban geometries are extended to another state. Normally, $\alpha \neq \beta$ is valid, which means that an element e cannot be transformed in itself, in order to build up an urban system. Additionally, $\alpha \to \{\beta_1, \beta_2\}$, and $\{\alpha_1, \alpha_2\} \to \beta$ are valid, because network design shape grammars are nonreversible. The stopping criteria is often related to budget, or space constraints in planning applications. The application of many rules r can result in an infinite set of urban transport systems. As an example, Table 1 proposes an context-free syntax R with corresponding elements E. R ignores external specifications, and therefore is called context-free (e.g. Friedman *et al.* 1992).

Table 1: Example context-free syntax R with corresponding set E of defined, generic road and intersection elements e.

Formal rule	Description
Vocabulary E ={e_1, e_2, e_3, ...}	
e_1	*Arterial road*
e_2	*Access road*
e_3	*Local road*
e_4	*Right of way junctions*
Context-free syntax R ={r_1, r_2, r_3, ...}	
r_1: $e_1 \to e_1 + e_1$	Network connectivity requires arterial roads to connect to other arterial roads.
r_2: $e_1 + e_1 \to e_1 + e_1 + e_2$	Arterials can be joined with an access road if a connected arterial network is maintained.
r_3: $e_2 + e_3 \to e_2 + e_3 + e_4$	An access road connected to a local road requires a right of way junction.
r_4: ...	

2.3 Shape Grammars

The definition of shape grammars resembles the definition of syntax. However, certain differences add up. E.g. Chomsky (1959, p.113) stated, that "no finite-state Markov process" can serve as a grammar of English. As in computer sciences, the rules can generate outcomes which are meaningless. Therefore, we claim that the syntax alone is insufficient to generate a reasonable outcome, e.g. for an urban planning environment. We aim at the properties of grammars in this section, and seek to define "grammars" as encompassing as necessary, ultimately for planning applications.

Multiple distinct definitions of grammars exist in linguistics. The distinct definitions approach syntax, language, and semantics in one or the other way. Scanning recent literature about grammars discloses an ongoing debate even in linguistics. The definition of shape grammars for transport and urban applications can not derived directly from linguistics. Therefore, a general definition of shape grammars is proposed for urban and transport planning, contextualized, and justified based on the current literature about grammar in cognate fields.

We assume in the following that shape grammars are applied in an urban and transport planning context. Moreover, we assume that the planners act rationally, and follow a certain overall intention, e.g. a sustainability goal or cost minimization, which are explicitly defined, or implicitly followed. We deploy the expression "objective" to define the intention in a qualitative or quantitative manner. Additionally, we suppose that planners act in a spatially defined area, called "site", which they intend to change directly, or indirectly, through structural changes.

It is obvious that a rule is limited to a certain purpose or meaning. Certain rules are designated for a specific context. Application specifications can describe the required environment to apply the rules. E.g. housing construction in tropics requires other rules, compared to moderate climates. The environment can include adjacent infrastructure, or global components like weather, social parameters, etc.. Therefore, we claim that, beside the syntax, grammars moreover include application specifications. Specifications are equivalents for semantics. The application specifications are valid for one specific rule, and therefore contrasts the "site" definition above, referring to the planner's view. The inclusion of application specifications differs from the definition of context-free grammars (Chomsky 1956, 1959). Applying the same rule with different application specifications might lead to a different outcome. Therefore, Figure 1 subdivides grammar in syntax as a rule set, and semantics as corresponding application specifications.

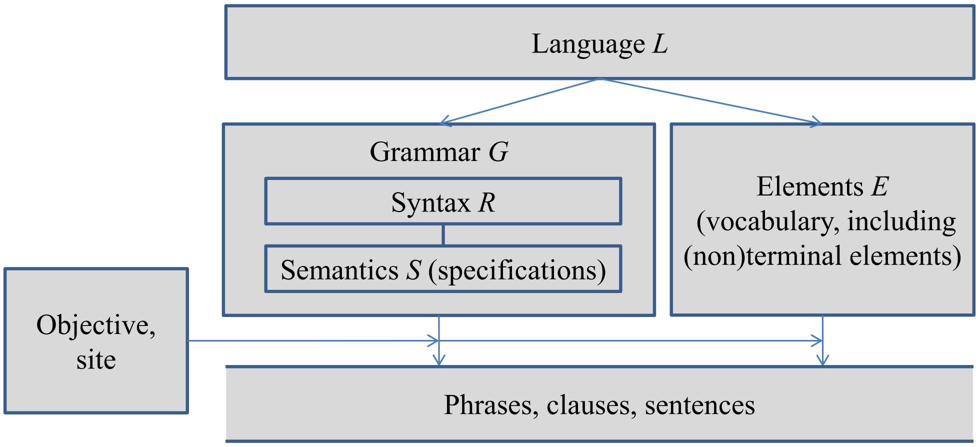


Figure 1: Contextualized language setup for shape grammars with exogenous planner's objective.

Supplementing the definition of shape grammars, "shape" can be defined in a rather technical way. From a geometric perspective (e.g. Lord and Wilson 1984), shape includes a set of marks, with position and orientation. We claim that "shape" refers to the urban, and network design context. "Shape" is slightly ambiguous in the current context, due to the fact that grammars not only apply to physical shapes, e.g. a certain functionality might be determined with a grammar rule too. Still, "shape" is widely understood in the urban planning context.

Regarding the application of shape grammars, key advantages are listed below. They are further discussed and emphasized throughout the paper.

- Shape grammars are applied to simulate urban growth (e.g. Vanegas *et al.* 2009; Weber *et al.* 2009), urban redesign (e.g. Bramley and Power 2009; Yerra and Levinson 2005), changes on the demand side (e.g. Dutton 2000), or even at new technologies (Geddes 1939).
- Shape grammars of various disciplines can be joined for an even larger set of grammars available for interdisciplinary applications (e.g. Alexander *et al.* 1977; Haas 2008). The multidisciplinary potential contrasts the distinct languages of the relevant disciplines. The distinct languages are often too sophisticated to interact with each other. The distinct disciplines often deploy methods and models, which are unable to be merged with methods of other, even cognate disciplines. Shape grammars overcome this interdisciplinary complexity by their straightforward definitions. The multidisciplinary rules potentially allow accomplishment of encompassing tasks, e.g. energy demand and supply (Keirstead and Shah 2011).
- Practitioners prefer robust and reliable methods. Shape grammars can satisfy these requirements and still remain adaptive to different scenarios (Marshall 2005; Weber *et al.* 2009).
- The application of shape grammars requires low computational costs (Parish and Müller 2001; Watson *et al.* 2008) and can be implemented in interactive planning tools (e.g. Jacobi *et al.* 2009; Weber *et al.* 2009). Therefore, grammars contrast spatial optimization, such as bi-level network optimization, in computational requirements.
- Deeper understanding of the structure of urban systems enhances overall urban planning, and the understanding of urban guidelines (Hillier *et al.* 1976; Michie 1974). This contrasts complex "blackbox" and cumbersome mathematical optimizations, and corresponding results restricted to a specific site.
- Grammars are independent in space and time. They are applicable at different planning sites. A rule set allows to retain an urban vision for longer time (e.g. Alexander *et al.* 1977; Geddes 1939), for long-term strategic master

planning. Planning authorities might change over years, but the detailed grammar descriptions allow to bequeath planning rules to future generations.

- Urban economics, transportation, and energy supply form complex decentralized systems. Knowledge about efficient leverage effects are essential for a successful design. Moreover, bottom-up rules like shape grammars are often accepted regulation methods.

Disadvantages of shape grammars:

- The effectiveness of shape grammar rules is often unknown in urban planning applications. Especially the assessment challenges the definition of grammars. The assessment requires a deeper understanding in the corresponding fundamental urban processes. E.g. network design rules often lack a systematic evaluation, e.g. cost-benefit-analyses, and do not remain explicit in their recommendations.
- The vast majority of research results are not formulated in shape grammar notation. We lack of shape grammar formulations, despite the broad expertise in the distinct planning disciplines. However, a potential transformation of the existing expertise and results into shape grammar rules would allow to enlarge the rule sets considerably, and simultaneously exploit the enormous potential for various grammars applications.
- Grammar rules can change over time. Rules might become impractical in the future, due to e.g. technological and behavior changes.
- Rules are often based on a common discretion, human perception, and aesthetic preferences. The validation especially of subjective rules might be ambiguous.

3 Shape Grammar History

Table 2 lists selected cognate disciplines applying grammars, referring firstly to the field of linguistics and computer science as early milestones in grammar evolution. The subsequent achievements evolved in parallel or consecutively and are described below.

Grammars are systematically applied in *linguistics*. Chomsky (1956, 1959) has been one of the first contributors on formal grammars. A formal language is defined as a language L, independent of its field or origin, of an infinite set. However, the language and structure of L can be investigated through the study of the finite devices, which are the grammars G, and which are capable of enumerating its sentences (Stiny and Gips 1972). This linguistic definition of the shape grammar language $L(G)$ has been modified, and transferred to many other fields.

Mathematics and especially *logic* employ grammars at an early stage. Logic defines an alphabet, which consists of terms, symbols, and variables. Formulations follow defined rules. E.g. "=" is defined as the standard equality, that both sides of a formulation are equal. Sentences can be generated, following the rules and alphabet above. At this point, it is interesting to note that Fagin (1974) subdivided all sentences of logic, especially complex problem formulations, in two distinct classes: $\mathcal{P}$ and $\mathcal{NP}$ (Non-deterministic Polynomial-time hard). This classification subdivides the sentences according to their complexity (Garey and Johnson 1979; Zimmermann 2008). If n is the problem size, and $\mathcal{O}\big(f(n)\big)$ the calculation costs, $f(n)$ is a polynomial function for all problems in $\mathcal{P}$. Design problems, e.g. network design, are often $\mathcal{NP}$ (Johnson *et al.* 1978) and solvable only in exponential time, which affects the transport and spatial planning considerably, because $f(n)$ is exponential in $\mathcal{NP}$.

OR (operations research) and *artificial intelligence* profit from optimization rules to solve complex problems, especially in (meta-)heuristics. An example can be given by Goldberg (2002). The well-known building blocks are defined clusters in a genetic code, similar to genes in a genome. Instead of recombining single elements of the genome, highly fit clusters of multiple elements are recombined to improve efficiency. Coates (2010) takes up on this idea and defines buildings as clusters. Vitins *et al.* (2012) implemented an evolutionary algorithm, and enable exchanges of clusters in transport networks. In *OR*, the notations ("max" / "min" and "subject to") are equivalent to the objective and the site definition in Section 2.3.

Computer sciences implement precisely-defined syntactic structures in various applications. Here, we want to emphasize that the syntax alone is insufficient for a working code. Interpreters are required to perform the actions indicated in the code. Semantics analyze the meaning of pieces of codes. Errors can still occur even with a syntactically correct code, e.g. null-pointer exceptions. Additionally, a working code does not necessarily fulfill the requirements of the user, and is not meaningful per se. E.g. cellular automata (Wolfram 2002) describe rules to continue from a starting or intermediate state to a consecutive state; however, they might not pursue an overarching goal. Therefore, stand-alone rules are inefficient, which also holds in planning applications.

In *geometry*, Stiny and Gips (1972), Stiny and Mitchell (1978, 1980) remain influencial. The geometry-based languages can be used for geometric art objects such as paintings, or sculptures. The application fields outside geometry range from procedural modeling, evolutionary and growth processes to conceptual design and aesthetic and visual arts. Beside many other geometric applications, Prusinkiewicz and Lindenmayer (1996) proposed the L-System, which consists of grammar rules and an alphabet of symbols, making larger and more complex system possible through recursion, such as plant morphologies.

Various authors have contributed to grammars in architecture, urban planning and transportation at the same time due to overlapping design aspects. We assign the achievements to the most relevant disciplines.

In *architecture*, the seminal contribution of Alexander *et al.* (1977) applied grammar principles to the languages of architecture and urban planning. The pattern language of Alexander *et al.* (1977) consists of a vocabulary including settlements, buildings, elements of the buildings and therefore varies in scale and covers both architecture and urban planning. The grammars of the language describe which elements of the vocabulary are more desirable and which combinations are inadvisable. March (1976) assign geometric design of buildings to an elementary boolean code, including elements and operations. Grammars are increasingly used in visualization of buildings and in film industry (Parish and Müller 2001; Vanegas *et al.* 2010). The methods of Stiny and Gips (1972) and Stiny and Mitchell (1980) are also adapted for design and construction purposes.

In *urban planning*, Sorkin (1993) and Cowan (2002) developed guidelines and prescriptions for general urban development in a qualitative way. Their work can be related to the movement of New Urbanism (Dutton 2000; Haas 2008; Mehaffy 2008). Following up on the idea of New Urbanism, a new generation of codes are developed for urban design. Smart Code (Duany *et al.* 2009) is a rule set, which incorporates all scales of urban planning, and which is applied in multiple neighborhoods in the U.S. and worldwide. Sustainable Street Network Principles (CNU 2012) were developed in a CNU project for transportation reform, and contributes in the field of New Urbanism. The focus is on walking and improved pedestrian infrastructure, and other modes of transport. It additionally focusses on the protection of the environment and ecology. A growing number of software solutions apply shape grammars for urban simulations (Section 4). The well-known space syntax comprises a set of methods and techniques to analyze spatial arrangements (Hillier and Hanson 1984; Hillier *et al.* 1976). Space syntax mainly analyses spatial configurations and arrangements, e.g. network or urban graphs (Gil and Read 2012). The analysis often represents a top-down approach.

In *transportation*, multiple norms and guidelines propose network design recommendations (AASHTO 2004; FGSV 2008; IHT 1997; VSS 1994). However, already in an early stage, LeCorbusier (1955) applied a strong hierarchical approach to city planning, referring therefore to a rule based approach. Without referring to a language, he suggested a hierarchical approach for road network design. The idea of a hierarchical approach is implemented in different standards of western countries. Alexander *et al.* (1977) contributed on road network layouts. Marshall (2005) introduced shape grammars by defining relationships between network element types without presupposing any particular final form. van Nes (2003) and Yerra and Levinson (2005) followed up on the hierarchical network layout and specified spacing, hierarchies, economic impacts and additional aspects.

4 Taxonomy

This section narrows down the broad view of Section 3 to urban and transport planning, and provides a systematic overview over the existing set of shape grammar rules.

Existing shape grammar classifications for urban planning and transportation can be found e.g. in Alexander *et al.* (1977), and Marshall (2005). Drawing on the broader existing literature, shape grammars can be assigned to divisions and classes, summarized in Table 3 for transport networks, and Table 4 for urban planning. Various classifications are possible for shape grammars taxonomy. We propose a function-based classification to address the purpose of each grammar. The divisions include *geometry*, *composition*, and *investments and regulations*, and subdivide the entire shape grammars set, for both urban and transport planning. More detailed classes, compared to the divisions, describe the aim of the rule more specifically. Table 3 and Table 4 serve as an overview over existing shape grammar rules. They can be extended with more examples, and additional classes.

A list of urban elements is required for completion of the language definition (Figure 1). However, we skip detailed descriptions and classifications of the elements of the urban shape grammar language due to lack of space. Various sources exist for network elements, e.g. Alexander *et al.* (1977). As an example, a potential classification in urban planning might include "roads", "tracks", "blocks", "zones", "landscapes" and "focal points" as elements of an urban design language (Lynch 1960).

An increasing number of software applications exist for shape grammars. This underlines the importance of shape grammars in urban and transport planning. Beirão (2012) developed a set of tools for combining design patterns, as part of the project City Induction (Duarte *et al.* 2012). It allows to compose an urban solutions for neighborhoods

Table 2: Major cognate fields of shape grammars applications.

Field	Elements (vocabulary)	Grammars	Results	Exemplary sources
Linguistics	Words out of characters, punctuation	Phrase structure, grammar rules	Text	Chomsky (1956, 1959)
Computer Science	Objects, instances, abstraction, inheritance	Methods, algorithmic procedures, cellular automata	Software program	Abady and Gardelli (1997), Wolfram (2002), Batty (2005)
Geometry	Alphabet of symbols and shapes (Point, edge, ..., n-dimensional object)	Algorithmic procedures, geometric transformations, structural relationships	n-dimensional shape	Stiny and Gips (1972), Stiny (2000), Lord and Wilson (1984), Prusinkiewicz and Lindenmayer (1996)
Architecture	Building Blocks	Standards, structural rules, zoning plans, function based rules, aesthetic rules	Building, structure, extensions, corrections	Alexander *et al.* (1977), Mitchell (1990), Wang and Duarte (2002), March (1972), March (1976), Coates (2010), Yazar and Colakoglu (2007)
Urban Planning	Buildings, public and private areas, parcels, neighborhoods	Standards, handbooks, zoning plans, function based rules, aesthetic rules, urban codes	Urban layouts, city plans, neighborhood design	Stiny and Mitchell (1980), Sorkin (1993), Cowan (2002), Duany *et al.* (2009), Lehnerer (2009)
Transportation	Roads, pathways, intersections, lanes, tracks, lines, vehicles, stations, stops	Standards, handbooks, safety rules	Transport networks and supply for different modes	Marshall (2005), Yerra and Levinson (2005), van Nes (2003), (AASHTO 2004)

Table 3: Classification of existing transport shape grammars for road and public transport networks.

Divison Class	Description	Exemplary sources
Geometry:		
Angle	Angle of adjacent road types,	Vanegas *et al.* (2009)
Loops	Circuit or cell-based road and line alignment, size of circuits	Cardillo *et al.* (2006), Levinson and Huang (2012)
Densities	Number of arms for intersections, dead ends, number of lanes for road types	Alexander *et al.* (1977); Vanegas *et al.* (2009)
Curvature, slope	Curvilinear design, steepness	Weber *et al.* (2009)
Composition:		
Connectivity	Connected elements, e.g. connected freeway or high speed rail	AASHTO (2004)
Function	Adjacent land use and building types, road access, parking, toll cordon.	Marshall (2005), Dutton (2000)
Hierarchy	Hierarchical road and intersection type distribution, Hierarchical service type distribution, stop densities, service frequency	Marshall (2005); Weber *et al.* (2009), Gil and Read (2012), VSS (1992), FGSV (2008), Marshall (2005)
Variation	Irregularity and variance in design (e.g. in old town vs. in uniform grid).	Alexander *et al.* (1977)
Investments and regulations:		
Density	Total road length, total number of intersections, block size, parallel roads, stop intervals	van Nes (2003), Vitins *et al.* (2013), Levinson and Huang (2012), Levinson *et al.* (2012)
System	Transport modes	Geddes (1939), van Nes (2003)

relying on a rule-based approach. Caneparo *et al.* (2007) describe a tool for building and neighborhood scenarios, incorporating rules in the design process, as well as evaluations of the suitable candidate solutions. CityEngine (ESRI 2012) and UrbanCanvas (Synthicity 2013) are commercial tools, and both rely on procedural 3D modeling, whereas the first build on GIS data, whereas the latter on transport and land use models. Grammar-based 3D modeling is also applied interactively for participatory design of planners and stakeholders (Jacobi *et al.* 2009). Architecture-related software tools can be found, e.g. Yazar and Colakoglu (2007).

Table 4: Classification of existing urban planning shape grammars.

Division Class	Description	Sources
Geometry:		
Building shape	Footprint, 3D shapes, angles	Stiny (1985), Schirmer and Kawagishi (2011)
Parcels, neighborhoods	Assignment and design	Kaisersrot (2011), Lynch (1981)
City design	Assignment and design, land use, prices	Alexander *et al.* (1987), Lehnerer (2009), Batty (2005), White *et al.* (2012)
Constitution:		
Function	Building and neighborhood type	Kaisersrot (2011), Duany *et al.* (2009), Dutton (2000)
Material, and construction		LeCorbusier (1955), Stiny and Mitchell (1980), Heisel and Yitbarek (2013)
Investments and regulations:		
Density	Units, population, building mass and densities	Bramley and Power (2009), Geddes (1939), König and Müller (2011), Duany *et al.* (2009)
Ownership and social interaction	Public and private space	Copper Marcus *et al.* (1998), Lehnerer (2009), Dutton (2000), Mikoleit and Puerckhauer (2011)

5 Shape Grammar Assessment

5.1 Objectives and Application Specifications

Objectives of urban planning projects include one or multiple goals, e.g. with an economic, social or environmental focus. The intention might be increasing quality of urban live, ecology, economy, or a compound measure. Planners apply explicit and implicit rules to reach the objectives. We claim that knowledge about the outcome of an objective function enhance grammar rule applications, and might become a touchstone for reapplications. Therefore, we need to assess the distinct grammar rules regarding specific objective functions. Literature provides a variety of assessment tools with a generalized costs, economical and environmental focuses. Table 5 lists example objectives and assessments methods. Sensitivity analysis (Kleijnen 2008; Saltelli *et al.* 2008) can be applied at all methods, and complement Table 5.

Table 5: Potential assessments of shape grammar rules.

Methods	Effectiveness measure	Example sources
Cost–benefit analysis	Generalized costs, external effects	van Nes (2003), Vitins *et al.* (2013)
Sustainability measures	Sustainability	Gil and Read (2012), Duarte *et al.* (2012)
Empirical evaluation	Modeling historical development	Strano *et al.* (2012), Levinson *et al.* (2012), Stiny and Mitchell (1980), Strano *et al.* (2012)
Qualitative analyses	variable	Marshall (2005), Alexander *et al.* (1977)
Surveys	Behavior, acceptance	Bramley and Power (2009)

Compared to context-free syntax, grammar rules combine rules with semantics (Figure 1). The efficiency describes the intention of the grammar rule (Table 5). Moreover, the application specifications describe the potential area of application, or site. Three examples of grammar rules, effectiveness measures, and application specifications are provided in Table 6. The application specifications obviously limit the application range of the grammar rules. Reassessment, discretion, and expert knowledge might adapt existing rules for new application specifications.

Table 6: Three examples of objectives, grammar rules, effectiveness measures, and application specifications.

Purpose	Grammar rules	Effectiveness measure	Application specifications A
Private transport and transit network characteristics (van Nes 2003)	Road spacing and hierarchies, transit network characteristics	User costs and infrastructure costs	Model application with lanes, headways, roads etc.
Social sustainable living (Bramley and Power 2009)	Urban density planning	Social sustainability	Survey of English Housing, Census of Population (England and Wales)
Transport network design (Yerra and Levinson 2005)	Hierarchical street design	Link–based revenue	Road infrastructure investment, small model application

5.2 Enhanced Choice Set Generation

Planners generate subsets, which are more appropriate in certain planning sites, compared to other subsets. The defined application specifications of grammar rules narrow the set of rules down to a well-defined subset. We aim at an optimized choice set generation to determine the most relevant grammar rule, in order to support planners' objectives. We propose to subdivide the effectiveness measure in two parts, the direction of the transformation, and their degree.

- The direction describes the transformations of the system related to an effectiveness measure. There might be a negativ, positive, or no effect regarding the objective.
- The degree describes the reaction of the system regarding the actual effectiveness measure. There might be a significant change within the system, when the system is in an elastic state. There might be a minor change in a inelastic (stable) system regarding the given objective.

This paper proposes marginal effectiveness and elasticities for the further specification of the effectiveness measure. Both measures are robust and accepted measures to assess the response of an observed variable. Both assess the changes of an outcome of an objective function related to the changes of an independent variable. In our case, dependent variable O equals e.g. the user costs, and the independent variable equals an underlying investment I change: $\left(\frac{\delta O}{\delta I}\right)$. In the context of shape grammars, marginal costs describe the efficiency of a specific rule regarding a given effectiveness measure in determined application specifications.

Elasticities ϵ are widely applied, and assess the changes of a dependent variable related to an independent variable. However, elasticities are free of units $\left(\epsilon = \frac{\delta O}{\delta I}\frac{\bar{I}}{\bar{O}},\text{ when assuming linearity}\right)$, facilitating comparison between different studies (e.g. Ewing and Cervero 2010). Recent achievements in survey methodology have enhanced elasticity estimations, aiming at e.g. more sophisticated urban and transport modeling (e.g. Goodwin *et al.* 2004; Hackney *et al.* 2007; Sanni and Albrantes 2013; Weis and Axhausen 2009). Table 7 provides three examples of elasticity calculations in urban and transport planning.

Potential applications of elasticities are energy consumption, emission, generalized travel costs, quality of urban space, satisfaction of residences (e.g. Bramley and Power 2009). The determination of the elasticities require a systematic data collection and processing. Data collection depends on the grammar, and the envisaged processing. An example is given in Section 5.3. A major drawback of many elasticities evaluations relate to the assumed underlying linear function

Table 7: Three examples of elasticities in urban and transport network design.

Scope of grammar	Efficiency measure	Investment I	Source
Hierarchical network	VMT	Intersection and street densities	Ewing and Cervero (2010)
Properties of planar graphs	Efficiency	Relative costs (densities)	Cardillo *et al.* (2006)
Road network investment (expansion)	Accessibility, travel demand	Speed, capacity, infrastructure cost	Weis (2012)

involved and the calculation of the average utility (Train 1986). Elasticities are calculated for mean values. We can under- and overestimate individual values. E.g. in value of travel time estimations, the linear model is outperformed by a more detailed function (Hess *et al.* 2008). We propose an approach to overcome some aspects of this issue in Section 5.3.4.

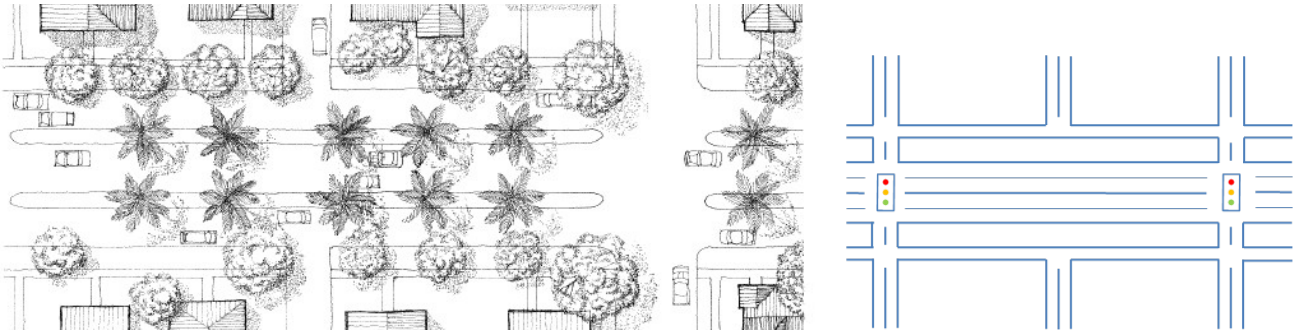
Sensitivity analyses uncover technical errors, find critical input variables, and determine model quality. Sensitivity analyses quantify and analyze uncertainty propagation, and study the uncertainty of the output of a model related to different sources of uncertainty in the model input (e.g. Kleijnen 2008; Saltelli *et al.* 2008). Additionally, correlation of the input variables influence the outcome (Embrechts and Hofert 2013). We emphasis that sensitivity analysis also enhance model understanding and model improvement.

5.3 Example Shape Grammar Assessment for Urban Boulevards

The following example illustrates the general methodological approach elaborated above about shape grammar assessments. The example encloses the design of boulevards deployed in larger cities around the world. We aim at an increasing understanding of boulevards, their characteristics, and potential changes in various (un-) congested network states. The impacts of boulevards are quantitatively estimated based on a current model and objective functions below. The current example bases on the effect of car travel; the effect on other modes can be derived similarly.

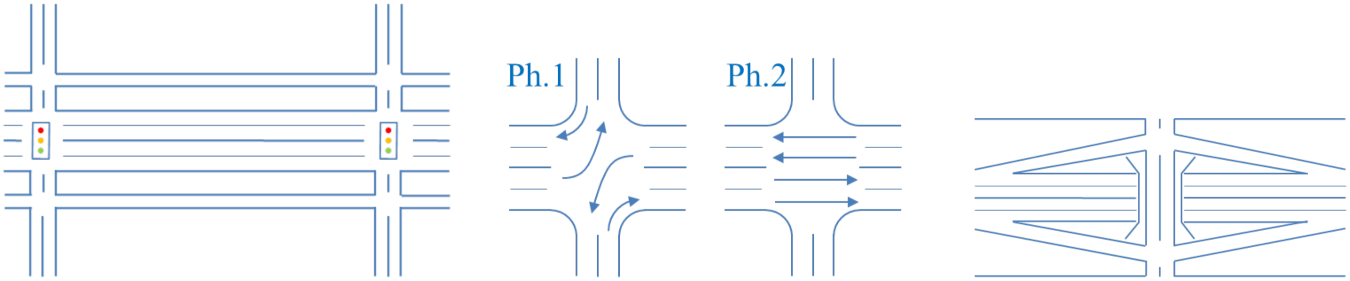
5.3.1 Objective and Application Specifications

The application specifications include a featureless plane to avoid biases due to historical design decisions, and terrain. The featureless plane is a square of either $2 \cdot 2$ or $3 \cdot 3[\text{km}^2]$ with a grid network of 100[m] bock size (400 or 900 blocks in total), similar to Yerra and Levinson (2005), Vitins *et al.* (2013), van Nes (2003). The modeled multiway boulevard is



(a) Sacramento, San Francisco Boulevard (Jacobs *et al.* 2002, p.197).

(b) Type 1, according to Figure 2(a), but with higher road capacities.



(c) Type 2, with signalized intersections (similar to Figure 2(b)), but improved pedestrian access without intermediate access roads.

(d) Type 3, with reduced number of conflict points at the center road intersection, with only two phases (Ph.1 and Ph.2), prohibiting direct boulevard crossing.

(e) Type 4, with multilevel diamond at the center road, requiring additional space, or construction on a lowered level.

Figure 2: Boulevard designs.

situated in the center of the network, and has a high-capacity two-way center road, and parallel minor one-way roads on both sides for access (Figure 2(a)). 4 boulevard types are proposed in the following differing in their intersection design for the major center road (Figure 2(b)–Figure 2(e)).

Type 1: Signalized intersections on the center road.

Type 2: Signalized intersections on the center road, and longer parallel minor roads for pedestrians and retail.

Type 3: Signalized intersections on the center road, but with reduced number of conflict points.

Type 4: Multilevel intersections of diamond shape to increase safety and efficiency.

The boulevard center line can be accessed and crossed every second block (similar to e.g. Arc de Triomphe in Paris, Passeig de Gràcia and Diagonal in Barcelona, Ocean Parkway in Brooklyn) in all Types, and contrasts the remaining network, with homogeneously distributed blocks, embedded in a grid network with right-of-way intersections. Right-of-way intersections are implemented due to low turn delays up to medium traffic volumes. We intended to include high-capacity roundabouts, similar to Paris; but the lack of consistent norms for high capacity roundabouts (>2 lanes) made evaluation impossible.

5.3.2 Objective functions

The objective is to assess the effects of a boulevard from a transport planning perspective. On the one hand, the boulevard either could reduce the overall network capacity due to turn restrictions and high signal delays. On the other hand, the boulevard can increase overall network capacity due to additional road length. Additionally, the costs can exceed user benefits, even over time.

We consider three objective functions to cover changes in user costs, spatial economics and external costs. User costs include monetarized travel time as a function of distance (Hess *et al.* 2008), including operating costs (VSS 2006c). Regarding spatial economics, we apply an accessibility measure, which is widely applied in transport and economics. It is based on Rice *et al.* (2006) who stated that doubling the working population proximate to an area raises productivity by ∼3.5%. Additionally, Porta *et al.* (2008) showed correlations between a centrality measure and commercial and service activities. Levinson and Huang (2012) summarized the theory of economies of agglomerations.

Accessibility is a proxy measure similar to the applied function in the above mentioned studies.

Accessibility A for location i is calculated as $A_i = \sum_{\forall j} \cdot X_j \cdot e^{-\beta c_{ij}}$ (Axhausen *et al.* 2008) with X_j as attractiveness of location j, β=0.2 and c_{ij}=travel time between i and j, applied in multiple Swiss studies. Again, an average density of population and jobs are used for attractiveness, the same as for travel demand determination (see below). The authors ignore land use dynamics, which also affect traffic flows, and only calculate the initial effect of a boulevard.

Regarding the external costs, we considered a set of variables (Table 8). Jacobs *et al.* (2002) evaluated safety of boulevard intersections. In general, boulevards seem not to be more dangerous compared to streets with comparable capacities or flows, according to an evaluation of empirical data (Jacobs *et al.* 2002). We therefore ignore safety changes due to the specific boulevard design. We exclude trucks, and public transportation in the calculation of external costs.

Costs for construction vary considerably in existing data and are often not available in detail. We extracted the costs from Litman (2011) and Alam *et al.* (2005). A major arterial costs about 1.3 [Mio.$/lane/km], a collector road costs 0.8 [Mio.$/lane/km] in a built-up area (year 2000), excluding land costs.

Table 8: External costs calculation for cars, excluding tax revenue due to additional gasoline consumption, according to the Swiss cost-benefit norms.

	Measure	Value	Source
Noise	Decibel	0.0140[sFr./veh.km]	VSS (2006b)
Air pollution	Particulate matter	$3.55 \cdot 10^{-2}$[sFr./veh.km]	VSS (2006b)
	Nitrogen oxide	$1.00 \cdot 10^{-2}$[sFr./veh.km]	VSS (2006b)
	Zinc	$1.30 \cdot 10^{-3}$[sFr./veh.km]	VSS (2006b)
Climate effect	CO_2 equivalent	$8.40 \cdot 10^{-3}$[sFr./veh.km] (values for 2010)	VSS (2006b)
Accidents	Roads	0.1741[sFr./veh.km]	VSS (2010)
	Signal light	0.1142 [sFr./veh.]	VSS (2010)
	Right-of-way	0.1697 [sFr./veh.]	VSS (2010)

5.3.3 Methodology

Travel demand is uniformly spread in the area considered. Every block is modeled as a demand generating point. Uniform demand distribution avoids a bias due to specific demand flows. The total travel demand is according to a dense four story perimeter block development in Zurich, and comprises all travel purposes according to the census (Swiss Federal Statistical Office (BFS) 2012). Demand flows vary to cover peak hours (Section 5.3.5). Further assumption, e.g. block size of 100[m], are explained in Strano *et al.* (2012), and are similar to e.g. Manhattan, Bogotá, or other cities.

A high resolution static model is deployed with detailed intersection delay calculations. The authors claim that intersection delays are crucial in urban areas. Both intersection types are calculated in detail according to the HCM (Transportation Research Board 2010). The demand assignment is conducted with a Frank and Wolfe algorithm (Frank and Wolfe 1956). Synchronisation of signals is implemented for the boulevard green wave. The network does not require other synchronisation. Equilibrium sensitivity analyses showed stable assignment results.

5.3.4 Effectiveness and Elasticities

Figure 3 depicts the apportioned travel distances and relative intersection delay related to increasing boulevard lengths. The reasons are threefold for increasing travel distances and decreasing relative intersection shares. Drivers remain longer on the boulevard, due to its higher speed, and low intersection delays (green wave). However, drivers also reroute

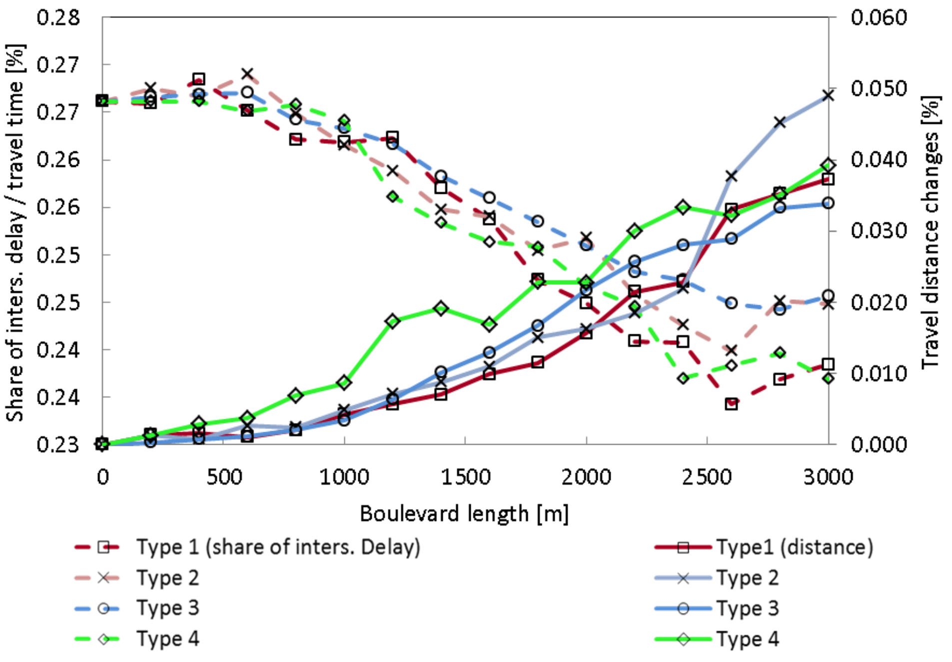
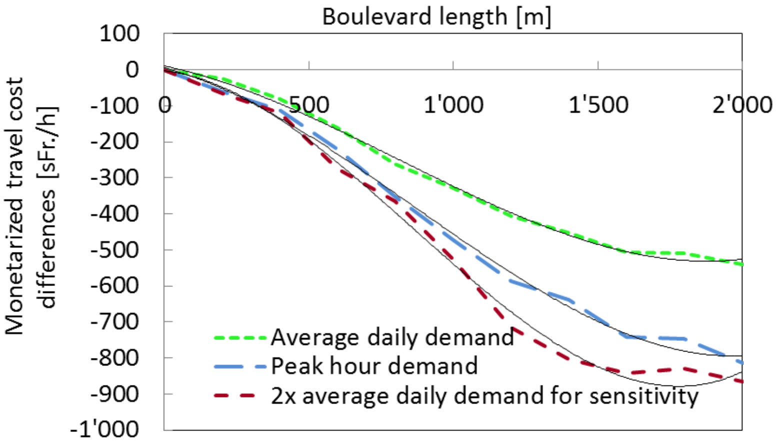


Figure 3: Average delays and travel distances for $3 \cdot 3$ [km^2] networks areas of all boulevard designs for peak hour traffic.

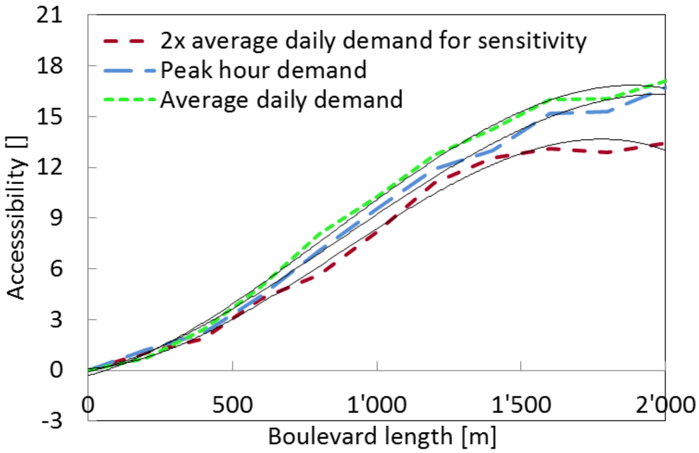
to avoid crossing the boulevard, due to higher intersection delays at the boulevard crossings. Additionally, asymmetric travel behavior is observed due to higher left turn delays on the boulevard compared to straight or right turn delays.

Figure 4 depicts the results of the evaluation according the three objective functions. Figure 4 indicates considerable trends for nonlinearity. Slopes decrease at short and long boulevards, which complicate elasticity calculations. However, Figure 4 also shows longer linear intervals, allowing elasticity calculation based on linear assumption. We approximate data with a polynomial function $f(l)$, to account for both effects. Polynomials with a degree of 3 are able to account for slope changes at both ends, avoid overestimation of the data points, and achieve high correlation (R^2). The highest slope values $f''(l_{max}) = 0$ are determined, based on polynomials of s-shapes like in Figure 4. We propose a two-sided application range $\{l_{min}, l_{max}\}$, in with values close to the highest slope values can be expected $\left(f'(l_{min}, l_{max}) = f'(l_{max}) \cdot \lambda\right)$. Therefore, we can calculate an approximated elasticity ϵ^s for the subset s of the data, which is inside the interval. Polynomials are approximated with the library in Apache Commons (2013).

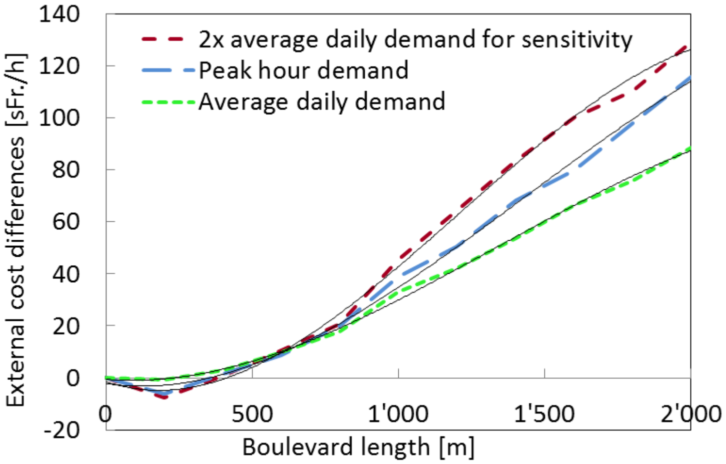
Table 9 summarizes the elasticities of the data for boulevard Type 1 with signalized intersections. ϵ^s is the elasticity within the application range. We have an increasing elasticity with higher flows, especially at the $3 \cdot 3$ [km^2] networks.



(a) monetarized travel costs.



(b) accessibility.



(c) external costs.

Figure 4: Total monetarized travel time savings, accessibility changes and external effects for different boulevard lengths of Type 1 (signal control) and network size $2 \cdot 2$ [km^2], including polynomial approximation (degree = 3).

Table 9: Marginal costs, application range, and elasticities for boulevards with signal lights (Type 1).

	Size of featureless plane [km^2]			
	$2 \cdot 2$		$3 \cdot 3$	
Marginal costs $\frac{\delta t}{\delta I}\left[\frac{sFr}{h \cdot m}\right]$ and application range [m], λ = 0.8	$\frac{\delta t}{\delta I}$ (lower – upper bound)	R^2	$\frac{\delta t}{\delta I}$ (lower – upper bound)	R^2
Average daily traffic volumes	-0.3970	0.9965	-1.067	0.9936
	(320–1’290)		(610–2’110)	
Peak hour traffic volumes	-0.5630	0.9958	-1.282	0.9766
	(370–1’370)		(970–2’270)	
2x daily traffic volumes (for sensitivity)	-0.7161	0.9933	-1.377	0.9006
	(440–1’280)		(1’010–2’240)	
Monetarized travel time elasticities $\epsilon^{s,T} = \frac{\delta t}{\delta L}\frac{\bar{L}}{\bar{t}}$ of	$\epsilon^{s,T}$		$\epsilon^{s,T}$	
average daily traffic volumes	1.285		1.312	
peak hour traffic volumes	1.292		1.719	
2x daily traffic volumes (for sensitivity)	1.371		1.864	

Additionally, the $3 \cdot 3$ [km^2] networks have higher values compared to the $2 \cdot 2$ [km^2] network, probably due to higher average travel distances. The minimum and maximum values of the application range increase at larger networks.

Table 10 compares different boulevard types, objective functions, and demand volumes n, enabling future recommendations. Boulevards with signal lights at the major center road (Type 1) have the highest travel cost and accessibility elasticities. Boulevards with longer minor roads (Type 2) have the lowest travel cost and accessibility elasticities. Boulevards with conflict free intersections (Type 3) and boulevards with multi-level intersections (Type 4) have lower elasticities compared to Type 1, values also differ in the size of the network. High R^2 are calculated throughout the evaluations.

Table 10 additionally summarizes the external cost evaluations. Multilevel boulevards have the lowest external costs elasticities. The remaining boulevard types have higher cost elasticities, but without a clear ranking order. Detailed evaluation showed that Type 1 and Type 2 have generally low external cost elasticities, but increasing values at increasing traffic flows. Overall, we observed that some external cost results might get approximated with other functions than polynomial functions of degree 3.

Table 10: Summary and comparison of the elasticities ϵ^s for $\lambda = 0.8$ and confidence of determination R^2 of all boulevard types n; with minimum and maximum ϵ^s values for different demand volumes n (average, peak hour, $2\cdot$ average for sensitivity).

	Size of featureless plane [km^2]			
	$2\cdot 2$		$3\cdot 3$	
Travel cost elasticities	$\min_n\{\epsilon_n^{s,T}\}, \max_n\{\epsilon_n^{s,T}\}$	$\min_n\{R_n^2\}$	$\min_n\{\epsilon_n^{s,T}\}, \max_n\{\epsilon_n^{s,T}\}$	$\min_n\{R_n^2\}$
Type 1: Signal	1.285, 1.371	0.9969	1.312, 1.864	0.9006
Type 2: Signal (pedestrians)	1.195, 1.256	0.9871	0.9425, 1.301	0.8442
Type 3: Red. of conflict points	1.133, 1.247	0.9948	1.302, 1.582	0.9945
Type 4: Multilevel	1.275, 1.328	0.9940	1.222, 1.354	0.9902
Accessibility elasticities	$\min_n\{\epsilon_n^{s,A}\}, \max_n\{\epsilon_n^{s,A}\}$	$\min_n\{R_n^2\}$	$\min_n\{\epsilon_n^{s,A}\}, \max_n\{\epsilon_n^{s,A}\}$	$\min_n\{R_n^2\}$
Type 1: Signal	1.314, 1.381	0.9930	1.360, 1.920	0.8970
Type 2: Signal (pedestrians)	1.197, 1.286	0.9863	0.8908, 1.292	0.8527
Type 3: Red. of conflict points	1.149, 1.246	0.9947	1.303, 1.611	0.9950
Type 4: Multilevel	1.281, 1.343	0.9937	1.271, 1.382	0.9898
External costs	$\min_n\{\epsilon_n^{s,E}\}, \max_n\{\epsilon_n^{s,E}\}$	$\min_n\{R_n^2\}$	$\min_n\{\epsilon_n^{s,E}\}, \max_n\{\epsilon_n^{s,E}\}$	$\min_n\{R_n^2\}$
Type 1: Signal	1.663, 1.901	0.9970	1.782, 2.917	0.9734
Type 2: Signal (pedestrians)	1.634, 1.900	0.9957	1.758, 2.509	0.9557
Type 3: Red. of conflict points	1.801, 1.827	0.9863	1.772, 2.212	0.9953
Type 4: Multilevel	1.494, 1.771	0.9973	1.252, 1.638	0.9787

5.3.5 Sensitivity Analyses and Evaluation

We identified multiple causes for increasing generalized travel costs in boulevard design, despite the high boulevard capacity. Three situations possibly trigger an undesirable network state.

- Drivers reroute off the boulevard onto parallel roads as soon as the speed decreases compared to the parallel routes. Rerouting decreases the original functionality of the boulevard and its center road. We found that rerouting off the boulevard increases average travel costs of all drivers up to 9% due to the unused capacity of the empty boulevard, and the congested boulevard crossings and parallel roads.
- Long boulevards separate urban space in two halves. This separation increases travel time considerably if the crossings have low speed or capacity. Especially the reduced number of crossings at boulevards (50% in our example compared to a regular grid) increases the chances of bottlenecks. Moreover, the feeder roads to approach the boulevard crossings get congested as well. Especially high intersection density of the feeder roads near the

boulevard increase delays additionally. Cars might drive around the boulevard to avoid congestion, if capacity of the intersections and feeder roads is insufficient.

- Drivers crossing the boulevard crossings increase waiting delay for cars driving on the boulevard, due to the adaptive signal and green time at the boulevards. This effect reduces overall travel time on the boulevard, and increase chances of rerouting on parallel roads.

Multiple solutions tackle the issues above. The boulevard axis needs to retain high flows during both uncongested and congested network states. Signals have longer turn delays due to the cycle characteristics. Therefore, road travel speed has to be higher on the center road due to the signals to maintain the desired high flows, compared to the parallel minor roads. Drivers might reroute onto parallel roads as soon as congestion is high enough on the boulevard, to reach an equilibrium state in their route choice. Therefore, capacity should be rather high at boulevards, to reduce delays and rerouting on parallel roads, shown in many boulevards worldwide (Jacobs *et al.* 2002). Similarly, free flow speeds on parallel roads might be reduced even more to prevent rerouting.

The functionality of a boulevard might be reduced through asymmetric demand, constructions, accidents, etc. High capacity signalized intersections reduce the chance of a breakdown of the complete boulevard axis. The capacity should be enough to supply a certain redundancy. Statistical analysis can estimate reliability and redundancy of the urban area considered (e.g. Bernard and Axhausen 2007). The evaluation showed that mainly the capacity of the signals are the limiting factor in boulevard design. Road capacity is not as limited, compared to signal capacity. Multiple and diverging approaching lanes can increase signal capacity for high flows.

Green waves reduce travel time on boulevards. However, we found out that green waves are not as influential as other infrastructure changes in the case of boulevards. Green wave only reduce uniform delay, which is low at high flows and adaptive green time. Therefore, total travel costs merely decrease when implementing green wave synchronisation.

We also conclude that the geographic location of the boulevard is critical within an urban area. Terrain barriers at the ends of a boulevard reduce the routes on the nearby boulevard and increases the on- and off flows at the intermediate intersections.

We ignored the share of long-distance through traffic, relevant for inbound or outbound traffic. However, we assessed the boulevard from a city perspective, and did not consider external influences like long-distance travel flow.

The following cost-benefit estimation holds for an approximately linear correlation for the boulevard length within the application range (see above) and the travel costs ($t_{absolut} \propto l_{Boulevard}$), and a low $\epsilon^{s,\,travel\,cost}$ value. The benefit to cost ratio $\frac{b}{c} > 1.0$ is reached after ∼8 years, when assuming the costs above, maintenance (VSS 2008), lifespan, and discounted costs (VSS 2006a), but ignoring land acquisition. For a time horizon of 100 years, the costs for land acquisition should not exceed 10'000 [sFr/m$_{\text{boulevard}}$] to reach $\frac{b}{c} > 1.0$. This low value is mainly due to discounted benefits, and repeating maintenance costs.

5.3.6 Shape Grammar Rules

Rule set for boulevard design:

Rule 1: Boulevards should have a certain minimum length to achieve considerable travel cost reduction and increasing accessibility. The minimum length is 2–3 blocks, only longer boulevards reduce travel costs, and increase accessibilities. ≥3 blocks for boulevards in larger networks, and higher average travel distance.

Rule 2: Boulevards with signals at the center road have the highest travel cost and accessibility elasticities $\epsilon^{s,T}$, $\epsilon^{s,A}$.

Rule 3: Travel speed on the boulevards has to be higher than on parallel roads, even when implementing signals with uniform waiting time.

Rule 4: The capacity of the center road has to be high enough to serve the flows. At least two center lanes each directions are feasible. Rerouting on parallel roads reduces generalized travel costs; e.g. parking spaces on minor parallel roads can reduce capacity and speed.

Rule 5: Boulevards reduce generalized travel costs of urban traffic if the major intersections at the center road provide enough capacity and low turn delays for the required flows. This holds also for the crossings of the boulevard, which might be bottlenecks for crossing traffic. At least 3 approaching lanes for the major intersections are feasible.

Rule 6: The following elasticities are estimated for boulevards with signals at the center road:

- $\epsilon^{s,\,travel\,cost} \simeq 1.3 - 1.9$, with higher $\epsilon^{s,\,travel\,cost}$ values at higher flows (e.g. peak hours).
- $\epsilon^{s,\,accessibility} \simeq 1.3 - 1.9$, with higher $\epsilon^{s,\,accessibility}$ values at higher flows.
- $\epsilon^{s,\,external\,costs} \simeq 1.6 - 2.9$, with higher $\epsilon^{s,\,external\,costs}$ values at higher flows.

Rule 7: Boulevards generate a $\frac{b}{c} > 1.0$ only if the land prices are low for acquisition. Obviously, boulevards as proposed above should be designed in an early stage of urban design. If not, additional economical estimations (e.g. Venables 2007) might be considered, compared to a standardized cost-benefit procedure.

Corresponding application specifications:

The above rules are based on the following assumptions and evaluations. The application specification is described in Section 5.3.1 and 5.3.3 in detail. We outline the major application specifications below:

- Static model is used for all evaluations with detailed intersection delay calculations (HCM, Transportation Research Board 2010).
- Urban density of 15'068 [pers/km^2] and 6'685 [jobs/km^2] are assumed, including peak hour and sensitivity flows. Travel behavior is based on the Swiss census (Swiss Federal Statistical Office (BFS) 2012).
- So far, the boulevard is modeled in a gridiron network, of $2 \cdot 2$[km^2], and $3 \cdot 3$[km^2], respectively.

6 Conclusion

This paper systematically reviews, classifies and highlights the existing literature on shape grammars of urban and transport planning. The comprehensive review reveals major articles and books published since the early and seminal results of Chomsky (1956). The contributions are classified according to their research fields, subjects, shape grammar objectives, and potential implementations. Moreover, synergies could be found between different languages, including logic, operations research, and linguistics.

Drawing on these achievements, this paper proposes an assessment methodology for shape grammars enclosing potential objectives functions, such as generalized costs, an economical measure, or external effects. The authors posit that assessment is inevitable for shape grammar rule development. The potential of shape grammars can be fully exploited in a broader planning context, when enhancing the effectiveness of grammar rule implementations through the proposed elasticity evaluations.

We successfully provided an example on how to assess a certain shape grammar rule embedded in urban design. This paper demonstrates that elasticities are particularly instructive for future real-world applications, also due to the enhanced understanding of the rules. This paper aims at increasing and more effective shape grammar applications, and hopefully will nurture further contributions about grammar assessment and application in planning.

Future research work can take three directions. First, research might be required for future interdisciplinary assessment methodologies to cover the complex urban system as a whole. Second, our model is tested on a gridiron network so far. Other assessment methods could include different topologies, and demand patterns, as well as stochastic assignments, or large-scale microsimulation. Third, assessment results can be compared with real-world application and data.

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