Evaluation and Reliability of Shape Grammars for Urban Planning and Network Design

Basil J. Vitins and Kay W. Axhausen

Abstract Shape grammars are increasingly applied in urban simulations and are promising tools for urban design, e.g., in procedural modeling. Shape grammars are interdisciplinary, straightforward, understandable planning tools, which potentially have the chance to overcome the complexity of urban design. Applications of grammars and rule-based methods could be found in cognate fields of architecture, urban and transport planning, geometry, and also in mathematics, and computer sciences.

Applications of shape grammars are described exemplarily for various fields of science. It is found that often a profound evaluation of the effect of certain shape grammar rules is missing in literature. Therefore, a set of four approaches is proposed in this chapter (1) to establish theory of grammars and rule-based approaches, (2) to evaluate existing shape grammar rules, (3) to generate and extract new shape grammars, and (4) to verify complex system wide shape grammars. The four approaches are exemplarily applied in the field of road transport network design; however, migration to urban planning and architecture seems promising for future applications. Reliability is addressed in the evaluation methods to tackle uncertainties in planning.

Keywords Shape grammar • Rule • Intersection type choice • Urban Simulation • Delay • Reliability • Network • Optimization • Meshedness

1 Introduction

This chapter focusses on advances in the development of shape grammars for urban planning, in particular for transport network design. Network design and urban form strongly interact with each other. Shape grammars are applied in various cognate fields of urban planning and are potentially able to overcome the complexity and interdisciplinary in urban design. March [30], Stiny and Mitchell [38], Lehnerer

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[29], and Duany et al. [14] already applied shape grammars to a certain extent in cognate fields of network design such as architecture and urban planning. An overarching theoretical approach and evaluation methods are proposed in this chapter for shape grammars. The approaches and methods are verified in network design; however, it is claimed that they are also applicable in an urban planning and architectural environment.

There is clear evidence that our current society relies on efficient transport systems [e.g., 5, 46]. Scaling effects increase productivity and social welfare of our society. However, transport networks are complex and interdependent, e.g. with urban planning, architecture, information, and energy supply networks. Thus, society aims at a reliable and robust transport system to absorb changes in demand without major infrastructure changes. The importance of reliable networks has increased even more for transport systems due to their limited capacity in parallel with their crucial function as a backbone for our economic system. So, the current transport systems have to fulfill two major tasks: The system should be as efficient as possible, while at the same time reliable and robust to short- and long-term changes.

To fulfill these tasks, authorities and responsible organizations face major challenges due to the characteristics and complexity of urban networks. The first important characteristic is the decentralization in any given network topology. With some exception graphs such as radial trees with root nodes (hub-and-spoke network), or Cayley trees with root nodes and hierarchical levels [12], transport networks lack predefined hierarchies within the set of elements of a graph G, such as nodes (v) and edges (e), e.g. the definition of centralized objects [28]. A priori, v and e lack of an order or relevance, also, e.g., for reliability and robustness improvements [e.g., 17]. Peripheral or minor edges might be as relevant, regarding reliability and robustness, as centralized and major edges [e.g., 26]. The lack of order or relevance differs from many biological networks with tree graphs, described, e.g., in [47]. Biological networks, such as vascular systems in plants, generally are more affected at a disfunction of an edge e closer to the root node. Lämmer et al. [28] confirm these findings and state that for transport networks the "topological organisation is less obvious and a hierarchical structure similar to a Cayley tree is not found at all" [28, p. 95]. The second characteristic is the lack of decomposition of a given urban network into independent subnetworks (subgraphs G_n) similar to the decomposition of a mathematical formula, and therefore the potential reduction in complexity. This holds again for most graphs, except, e.g., radial graphs with root nodes. The second characteristic implies that network elements and their attribute (a_{v_n}, a_{e_n}) might depend on other attributes $\{a_{v_n}, a_{e_n}\} = f(\mathbf{a})$. An example of the potential dependencies of the attributes is the nonlinear problem of travel demand assignment under intersection delay considerations [33, p. 213]. Another example is the design and choice of road types, which might depend on urban densities. Overall, both characteristics might be obvious, but explain the vast complexity arising in network design.

Knowing about the ever increasing relevance of transport systems in conjunction with their complexity, plans and recommendations are required and expected for proper and improved transport system and urban designs. Recommendations are essential for planners, and authorities for settlement development. Moreover, recommendations can be bequeathed to future generations, adapted, and improved for future changes and increasing efficiency.Often, design recommendations can be formulated as rules. Rules are more adaptive in their application, compared to, e.g., predefined rigid design patterns. In the following, they are referred to as shape grammar rules as a general expression, which includes recommendations. The definitions of shape grammar rules for planning are relevant research tasks, due to the required interdisciplinary and fundamental knowledge, and potential universal applications of the new recommendations.

Following paragraphs sketch existing results on shape grammar rules in academia and practice. Drawing on existing academic literature, at least six major research directions can be identified with examples, in urban planning and cognate fields. (1) An architectural example is provided, e.g., by Stiny and Mitchell [37], who developed and applied grammar rules for the Palladian villa style. Lehnerer [29] described elements, building blocks, and corresponding grammar rules for building plans. (2) In transport network design, Marshall [31] focusses on streets and patterns and underlying grammar rules, e.g. for road type choice. Van Nes [44] evaluated and optimized road and public transportation networks and characteristics, such as road spacings and densities. (3) In urban planning, Duany [14] focussed on an urban code, related to the New Urbanism movement [e.g., 21]. Southworth and Ben-Joseph [34] presented qualitative designs for a functional, livable, and economic city. (4) In geometry, Stiny [36] implemented shape grammars for paintings and sculptures. Prusinkiewicz and Lindenmayer [32] proposed the L-System, which consists of rules and an alphabet of symbols, making larger and more complex systems possible through recursion, such as plant morphologies. (5) Mathematics and operations research have focussed on rules for a longer time. Logic defines an alphabet, which consists of terms, symbols, and rules, such as the "=" symbol. In operations research, e.g. evolutionary algorithms apply rules for optimization. (6) In computer science, an increasing number of urban models and simulations have been developed in recent years [e.g., 18, 43] based on procedural methods to simulate and visualize urban areas. The seminal books of Alexander et al. [3] and Alexander [2] cover aspects of (1), (2), (3), and can be seen as a cornerstone in the development of grammars, also for future development, maybe even in (6).

Beside academic literature, norms and design guidelines are well-known platforms for recommendations about network design for both transport and urban planning for practitioners and authorities. Network design guidelines [e.g., 1, 20, 22, 23, 52] discuss only some aspects of urban design and network topology. The "Urban Street Geometry Design Handbook" [23] focusses on road types and hierarchical design, conflict points at intersections, and intersection spacing. "A Policy on Geometric Design of Highways and Streets" [1] proposes hierarchical designs and a focus on technical design and road geometry. The "Planning and Urban Design Standards" [4] elaborates also on hierarchies and connectivity within network design. High connectivity is recommended for future city design and discussed for multiple urban network layouts. Both the "Urban Street Geometry Design Handbook" and "Planning and Urban Design Standards" contain elements of the new urbanism movement [e.g., 15].

Multiple advantages occur when applying a rule-based approach in urban and transport planning. Shape grammar rules can be efficiently applied by network designers and spatial planners without extensive transportation knowledge. Shape grammar rules require low computational costs and are more adaptive in their applications. They have the potential to overcome the complexity of transport networks. Moreover, shape grammar rules of various disciplines can be joined for completeness and for an encompassing design language.

Despite the various past developments and vast expert knowledge in network design, the consensus on "best practice" in urban network design is currently vague. It is found that also some of the planning recommendations are vague, while others merely rely on past examples and are incomplete. Often, recommendations for dense or congested urban road network design are not covered in the guidelines, even though networks are faced with high demands in urbanized areas. There is a lack of fundamental knowledge about the effect of different network designs. Especially the new urbanism movement proposes new design ideas, but often relies on qualitative data for research and practice. Recommendations on reliability and robustness improvements are often missing in the guidelines. A lack of knowledge is problematic especially due to the issues discussed above, namely the increasing number of applications and the complexity of network design. Especially, there is a lack of methodology in the actual development and evaluation of rules, despite their increasing applications in e.g., urban simulations.

This chapter contributes to the fundamental knowledge on urban design and shape grammars. It claims that quantitative evaluations of shape grammar rules shed more light on the effect of shape grammar rules. Therefore, theory on the language concept is elaborated for shape grammar rules as well as three approaches for quantitative evaluation and derivation of shape grammar rules. The approaches are able to crystallize potential grammars and evaluate rules for network design. Additionally, it could be found that during evaluation of shape grammar rules, more insights are gained for network design, serving as additional knowledge for future planning.

2 Applications of Shape Grammar Rules

Example applications of design rules can be found worldwide in new districts and growing cities. Many applications are due to the large ongoing urbanization around the globe and growing urban centers. Additionally, after natural disasters, such as earthquakes or tsunamis, economies and transport supply need to recover as quickly as possible. Beside designing new districts, existing urban areas are under constant changes regarding demand and supply. Existing areas have to address new or changing requirements in living or commercial areas, such as changing travel demand. In existing urban areas, authorities are urged to maintain the level of service in the light of changing requirements, and therefore have to rely on planning recommendations.

Researchers and practitioners have applied shape grammar rules in urban planning and cognate fields, sometimes to an increasing extent in recent years, e.g. in urban simulations. Some fields have overlapping aspects, such as transport network design, which are considered in both urban planning and transportation. Shape grammars are applied to simulate urban growth [e.g., 45, 56] or urban redesign [e.g., 9, 57]. Figure 1 shows examples of shape grammar rules, each corresponding to plans which are visualized in Fig. 2. Both Figs. 1 and 2 are explained in the following. An encompassing list of shape grammar rules and additional examples are provided in [48].

Figures 1a and 2a refer to rules available for transport planning and network design. The Swiss norm for transport network design [51] specifies a strong hierarchical design for road types, in which adjacent road links differ in one

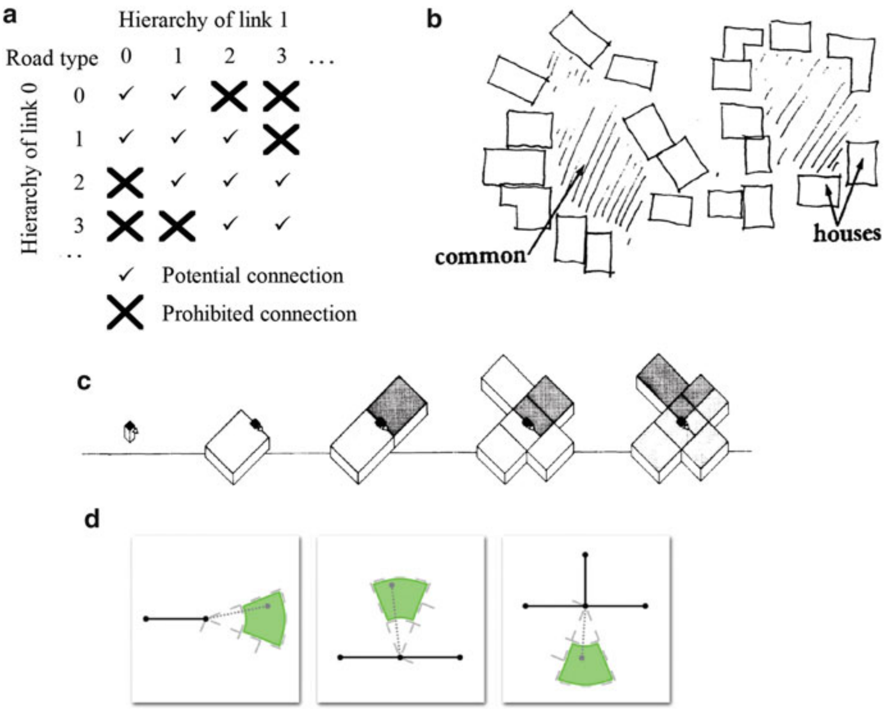

Fig. 1 Examples of shape grammar rules in transportation, urban planning, architecture, and computer science. (**a**) Shape grammar rule for hierarchical network design, adjacent road links should differ in one hierarchy at most. (**b**) Clustering of about 8–12 houses around some common land and paths [3, p. 202]. (**c**) The beginning of a prairie-style house, with a center fire place and rooms adjacent in a butterfly shaped composition [35, p. 13]. (**d**) The design and geometry of a new road element [56, p. 5]

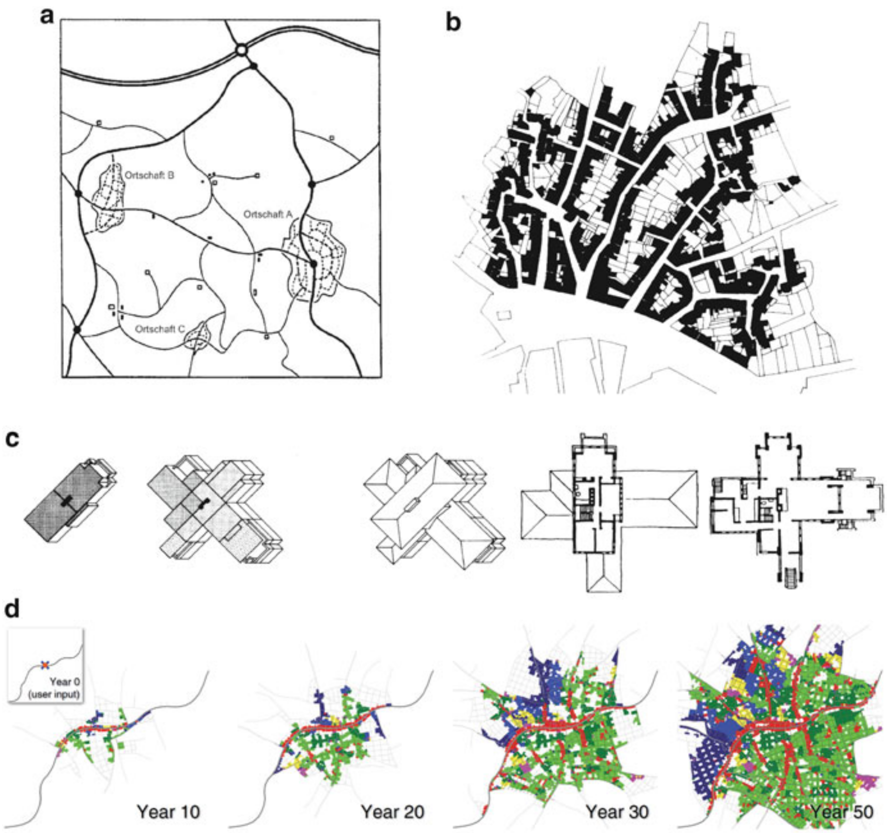

Fig. 2 Examples of grammar rule applications in transportation, urban planning, architecture, and computer science, related to the rules in Fig. 1. (**a**) The Swiss norm for road network design [51, p. 5]. (**b**) Example city design of Alexander et al. [3, p. 190]. (**c**) Prairie house from Stiny [35, p. 12] built on grammar rules. (**d**) Simulation of a city and its growth, including various land use types [56, p. 4]

hierarchy level at most (Fig. 1a). Figure 2a shows an example network of the current Swiss norm for network design. Figure 1b refers to urban planning and visualizes the recommendation of housing clusters around some common land, to support quality of the neighborhood, and increase comfort of the inhabitants. Figure 2b exemplarily shows a city quarter after the application of multiple rules of Alexander et al. [3], among others the clustering rule.In architecture, similar approaches are applied when comparing to urban planning above. Rules are defined for the design and construction of new buildings, such as the specification of rooms and their spatial relation. Figure 1c visualizes the design grammar of a prairie house, based on the ideas of Lloyd Wright [35], and Fig. 2c shows the final building and floor plan. In computer science, rule-based approaches are widely applied especially in programming languages. Grammar rules can be directly implemented in computer codes. Figure 1d shows an example schema which defines direction and length of new roads, related to the road design of the previous development steps. The outcome of this and additional grammar rules is displayed in Fig. 2d.

3 Potential Study Designs for Grammar Evaluations

This chapter improves the understanding of the effect of shape grammar rules. Different study designs are proposed and explained for the evaluation of shape grammar rules. Figure 5 shows a schematic overview of four proposed study designs, whereas study design 0 covers the underlying theory and theoretical requirements. Multiple study designs are proposed because shape grammar rules can be diverse in their content and complexity. Some rules describe the overall and general topology, like the T-junction rule of Alexander et al. [3], while others describe very specific designs, such as boulevards, which implies specific design on very limited space. Obviously, the definition and validation of one rule might differ from the validation of the other rule. Four study designs are proposed separately to overcome the diversity of shape grammar rules and corresponding verification problems.

3.1 Study Design 0: Theoretical Foundation

Study design 0 aims at a discussion of the theoretical aspects of shape grammars for a complete and sound definition of shape grammar rules for urban planning. The goal of study design 0 is to evaluate if the concept of shape grammar rules is sound and feasible in a theoretical and qualitative way (Fig. 5a), and that prerequisites for shape grammars and the necessary definitions are known. Study design 0 is required as a theoretical foundation, for further quantitative evaluation methods.

3.2 Study Design 1: Straightforward Application of Shape Grammar Rules

Study design 1 focusses on existing and defined shape grammar rules, which lack evaluations. In study design 1, shape grammar rules are applied and evaluated systematically in different network scenarios, and eventually compared with a reference scenarios without rules (Fig. 5b). This allows a quantitative comparison of scenario 1 (after implementation) and scenario 0 (before implementation). Therefore, evaluation can be conducted based on the differences between scenarios 1 and 0. The methodology of study design 1 is similar to standard scenario

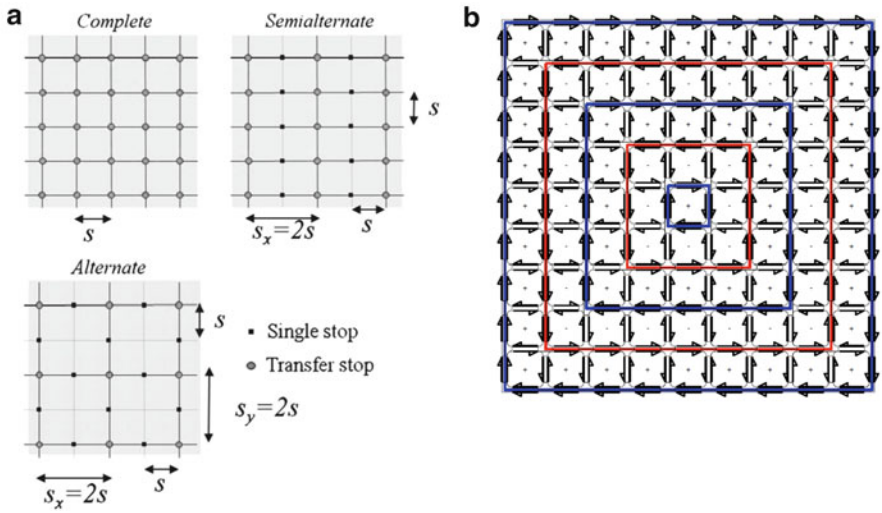

Fig. 3 Implementation of specific network rules, including evaluation, according to study design 1. (**a**) Public transport network design and different stop types [19, p. 940]. (**b**) Specific intersection types and turn restrictions [16, p. 10]

evaluations. However, study design 1 focusses primarily on rules, their effect, and the enhancements of rules. It is important that rules are not evaluated themselves as stand-alone rules, but their effects on networks or study areas.

Two examples of an application of study design 1 are provided in Fig. 3. Figure 3a refers to the design of public transport networks (also described in, e.g., [44]). Public transport networks consist of many variables, such as stop type and spacing, or line densities. Figure 3a shows three public transport networks designed according to certain rules [19]. The three networks are evaluated and compared regarding an objective function. Additional parameters, such as spacing, are evaluated as well in the mentioned research. Figure 3b refers to Eichler et al. [16], who determined specific intersection types and turn configurations, and evaluated network patterns in multiple example gridiron networks. Eichler et al. [16] applied study design 1 and evaluated the resulting network regarding travel distance changes. Both examples in Fig. 3 describe network conditions with implemented infrastructure changes. This allows a comparison between condition 1 (after implementation) and condition 0 (before implementation).

3.3 Study Design 2: Optimization and Derivation

Study design 2 extracts rules from already designed networks. Study design 2 is only applicable if it is known which networks are efficient and already optimized. Study design 2 assumes that optimized networks feature certain characteristics, which can be determined and extracted for future recommendations and shape grammar rules. Then, the designed and optimized networks are statistically evaluated regarding potentially significant characteristics. Multiple networks are needed to be evaluated for statistical analyses and significance. New rules are extracted and statistically justified under various transport conditions. Figure 5c shows a scheme of the proposed study design 2.

Figure 4 shows existing applications of study design 2. Figure 4a,b refers to spatial optimization methods, whereas Fig. 4c,d refers to statistical evaluations. Two examples for spatial optimization are provided in Fig. 4a,b. Figure 4a refers to the project [27]. The software developed in this project simulates and evaluates iteratively urban scenarios. The software is applied in parcel relocation and optimization, dependent on certain changeable parameters. The software designs and optimizes urban scenarios, but excludes grammar rule extraction. Figure 4b shows a network on a featureless plane with self-evolutionary structures, evaluation and

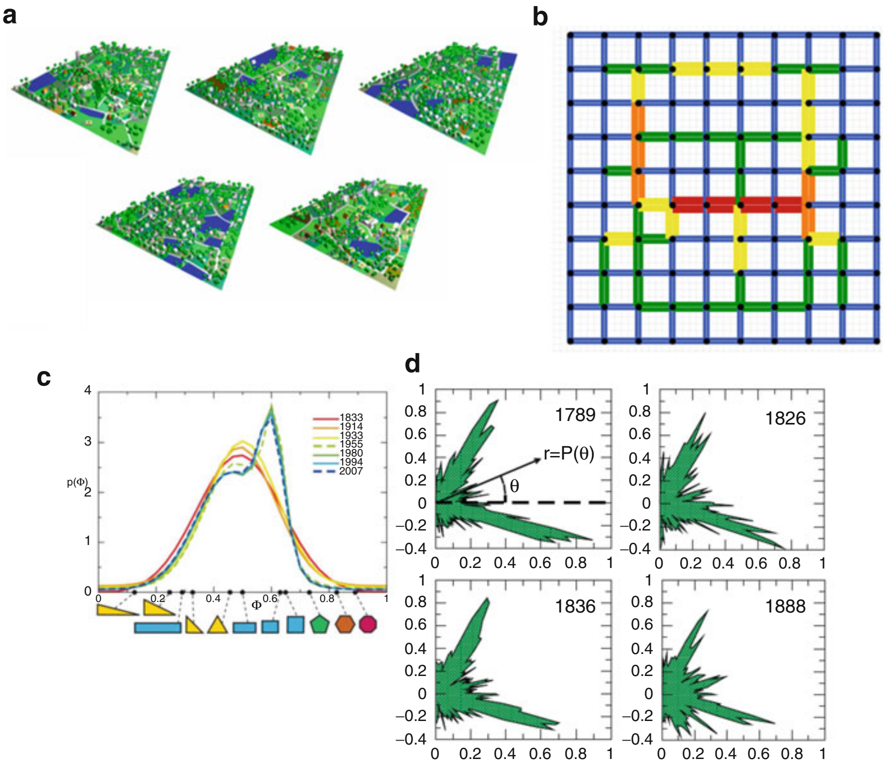

Fig. 4 Implementation and optimization methods for spatial planning. (**a**) The software Kaisersrot [27] generates and optimizes parcel distributions. (**b**) Self-evolutionary networks design and their evaluation, e.g., at evolution of ring structures [57]. (**c**) Distribution of parcels and its changes over time [39, p. 4]. (**d**) Angle distribution and historical evolution of the network of Paris [7, p. 6]

investment models for optimized road and intersection type choice. Both examples in Fig. 4a,b optimize scenarios based on infrastructure changes. In study design 2, it is additionally suggested that network characteristics are extracted from the optimized scenarios. Figure 4c,d depicts two example approaches from literature, each focussing on statistical evaluation of network characteristics. Figure 4c shows the evaluation of a real-world parcel shape distribution, based on multiple existing road networks. Figure 4d depicts the angle distribution of a descriptive evaluation.

Two issues potentially occur applying study design 2. First, the problem of increasing complexity hampers the design and optimization of large networks considerably. Therefore, reasonable assumptions are necessary to narrow down the vast search space size. Second, study design 2 lacks reference networks similar to study design 1. Therefore, approach study design 2 does only allow statistical analyses of a set of networks under consideration.

3.4 Study Design 3: Implementation and Comparison

Study design 3 focusses on more complex shape grammars compared to study design 1, such as road type choice and design (e.g., Fig. 1a). While certain shape grammars can be implemented and evaluated right away, other shape grammars are less concrete in their implementation (e.g., hierarchical network design). Especially rules, which affect the entire network topology, are more complex and study design 1 cannot be applied right away for evaluation. In study design 3, the shape grammar rules are defined first and then applied and implemented within a network design method. Network scenarios are designed and optimized based on the designated shape grammar rule and an underlying design method, e.g. as shown in Fig. 4a,b. Study design 3 proposes to apply shape grammar rules during the scenario design and optimization, and therefore contrasts study design 2.

Quantitative and statistical evaluation measures compare the network designs with a comparison set of reference networks designed without the designated shape grammar rule, or with a set of networks designed with a different shape grammar rules. Examples of network evaluations are given in Fig. 4c,d. Furthermore, the evaluations take places under various patterns, such as variable travel demand, to gain additional insights. However, it is emphasized that shape grammar rules are not evaluated in isolation but after application in different network conditions. Figure 5d depicts the comparison method of Study design 3 schematically.

3.5 Objective Function

Objective functions are needed for evaluation purposes in the proposed study designs methods (Sects. 3.2–3.4). This chapter focusses on shape grammar rules and evaluations for transport network design. Objective functions for transport planning

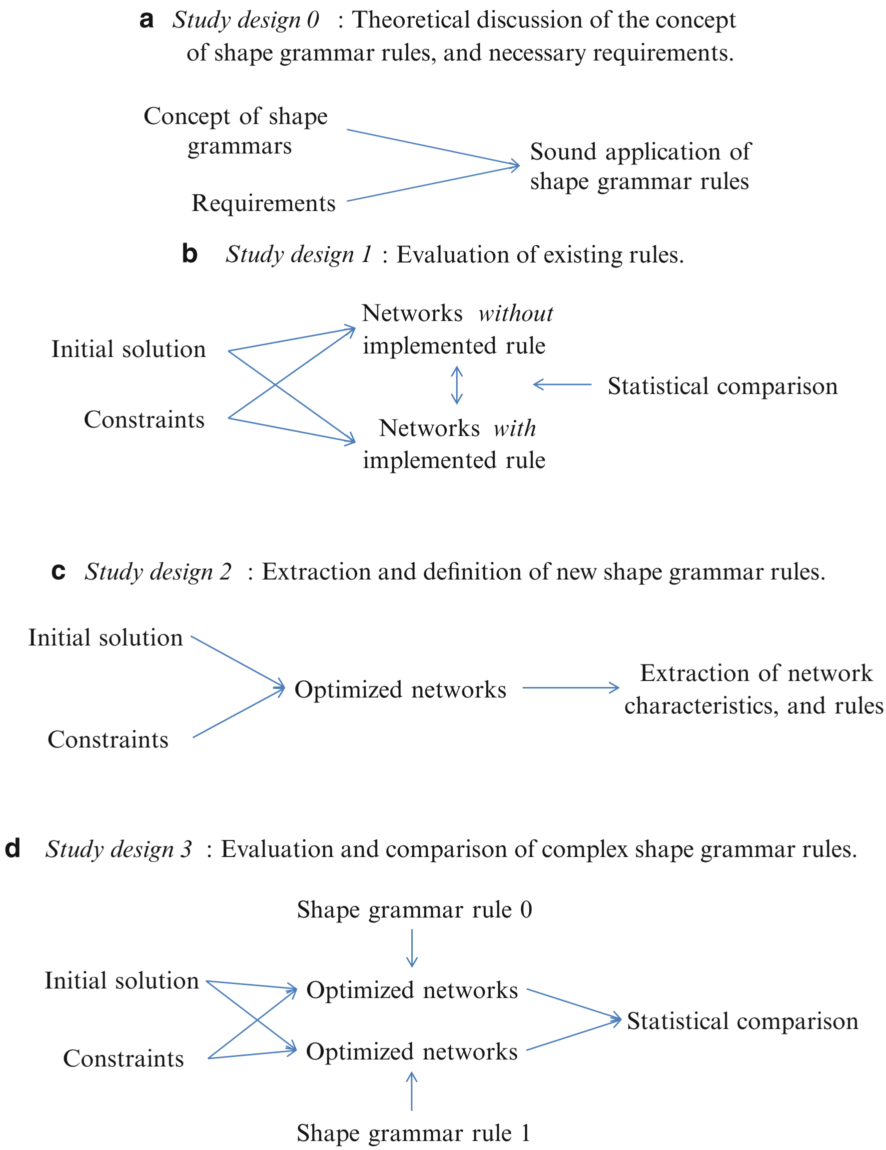

Fig. 5 Applied study designs to tackle research questions 0–3

purposes are relatively straightforward. However, evaluations can be extended with additional functions. For example, Bramley and Power [9] evaluated social sustainability regarding urban form and housing types.

Transport economics and guidelines for cost–benefit analyses (CBA) propose a large set of evaluation methods and corresponding parameters mainly for infrastructure changes [e.g., 54]. CBA considers user costs, infrastructure costs, and additional impacts discounted for a reference year. Focussing on the external effects, existing studies and norms define costs for noise, air pollution, and climate changes [53], or car accidents [55]. Also based on economics, accessibility A is a widely accepted and applied measure [e.g., 6]. A can be calculated for location i, for example, as $A_i = \sum_{\forall j} \cdot X_j \cdot e^{-\beta c_{ij}}$ with X_j as attractiveness of location j, $\beta = 0.2$ and c_{ij} = travel time between i and j.

Three objective functions are used in Sect. 4. The first function is based on the CBA and contains generalized travel costs. The second function contains accessibility based on population and job densities. The third function is based on the external costs, namely noise, air pollution, climate effects, and accidents.

3.6 Maximum Supply Approach

Multiple methods and measures exist for network reliability, robustness, resilience, etc. This section focusses on one aspect in urban planning, which is particularly uncertain and mostly varies over time: the level of travel demand. The uncertainty is due to gradually or unexpected changes in urban densities, e.g. population, jobs, leisure facilities, or changes in travel behavior due to cost or technological changes.

The proposed method especially accounts for unknown future travel demand (similar to, e.g., [19] or [8]), and shape grammar evaluation (Sect. 3). Therefore, the method assesses the maximum increase in urban density a network could cater for, without unacceptable travel time increases $\Delta d_{\text{structuraldata}}\left(\Delta t^{x\%}\right)$, when d is the urban density (jobs, population, ...) and $\Delta t^{x\%}$ the travel time difference. The focus is on travel time due to its importance in economic measures. An upper bound is assumed for travel time changes $\left(20\,\%\text{inthefollowing}, \rightarrow \Delta d_{\text{structuraldata}}\left(\Delta t^{20\,\%}\right)\right)$, which is achieved by gradually increasing urban densities. An average peak hour demand is defined for each density based on census data. Obviously, travels demand depends on other activities, daytime, mode share, car occupancy, which are ignored in this evaluation for simplicity. The aim is to determine how much urban densities can be increased with a given area and network design, but without an unacceptable increase in travel time. The major advantage of this approach is its focus on the relative difference and reduced dependence on transport infrastructure density, such as total lane length, as seen below in Sect. 4.4.

4 Study Designs for Road Network Design

Four study designs were described in Sect. 3 for the evaluation of different shape grammar rules. They are applied and discussed in the following.

4.1 *Language Approach for Urban Design (Study Design 0)*

This section focusses on the theoretical justification of shape grammars. Grammar $\mathcal{G}$ consists of rules $\mathcal{R}$, similar to a syntax. $\mathcal{R}$ are responsible for the "mechanics" of a certain language $\mathcal{L}$. For example, Stiny and Gips [36] described rules for geometric shapes and provided an abstract definition. However, it is stated that grammars do not only consist of rules. In addition to rules, $\mathcal{G}$ consists of semantics $\mathcal{S}$. In transport and urban planning context, $\mathcal{S}$ is basically responsible for all the information except the rules itself. In particular, $\mathcal{S}$ contains information about the effect of $\mathcal{R}$, such as effect on efficiency, safety, livability, etc. This is regarded as essential for the definition and improved application of rules. Moreover, $\mathcal{S}$ defines the application range in which the grammar rules $\mathcal{R}$ can be applied for reasonable design. For example, certain rules might be defined for urban environments, while others for rural environments.

It is assumed that the planners and designers act rationally and follow a certain overall intention, e.g. a sustainability goal or cost minimization, which are explicitly defined, or implicitly followed. The expression "objective" is deployed to define the intention in a qualitative or quantitative manner. Additionally, it is supposed that planners act in a spatially defined area, called "site," which they intend to change directly, or indirectly, through structural changes. Obviously, the phrases, clauses, or sentences are buildings, neighborhoods, or transport networks in the case of urban shape grammar applications. Figure 6 visualizes the proposed language approach. The two elements in the bottom of Fig. 6 refer to the application of shape grammars, and therefore include planners objective, site, and the resulting urban design.

Alexander et al. [3] showed an entire set of rules, and corresponding application and background information. Another example is the hierarchical network design approach of Marshall [31], which defines rules of how adjacent roads of the same or different type might be added to each other. In addition to these rules, adjacent urban land use types are defined for meaningful planning. Information about the context

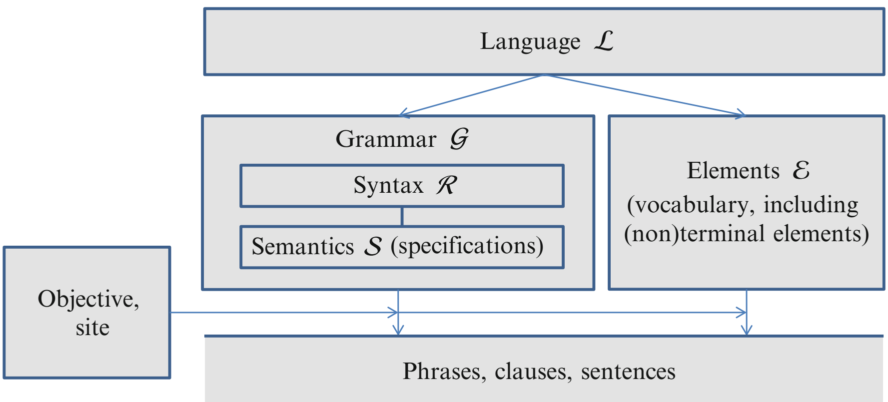

Fig. 6 Contextualized language setup for shape grammars with exogenous planner's objective

and effect of shape grammar rules seems reasonable for the definition of future shape grammar rules, and successful applications. These theoretical considerations additionally support the evaluation methods, which are presented in this chapter, and the applied underlying objective function (Sect. 3.5).

4.2 Example Shape Grammar Application (Study Design 1)

Here, a boulevard design is exemplarily evaluated for potential shape grammar rules on boulevards. For example, Jacobs et al. [25] describe multiple boulevards worldwide, with different designs. The initial boulevard design is sketched in Fig. 7a. It is implemented in a gridiron transport network model, with $100 \cdot 100$ [m^2] block size and standard road and intersection parameters [42]. Urban densities and travel demand are based on an average four-story perimeter block development of Zurich and the census data [40]. Vitins and Axhausen [48] elaborate additional evaluations and boulevard types, in addition to the design in Fig. 7a.

Figure 7b–d depicts the results of the evaluation according to the three objective functions (Sect. 3.5), depending on various boulevard lengths l. Figure 7b–d indicates overall nonlinearity of the functions. Data are therefore approximated with

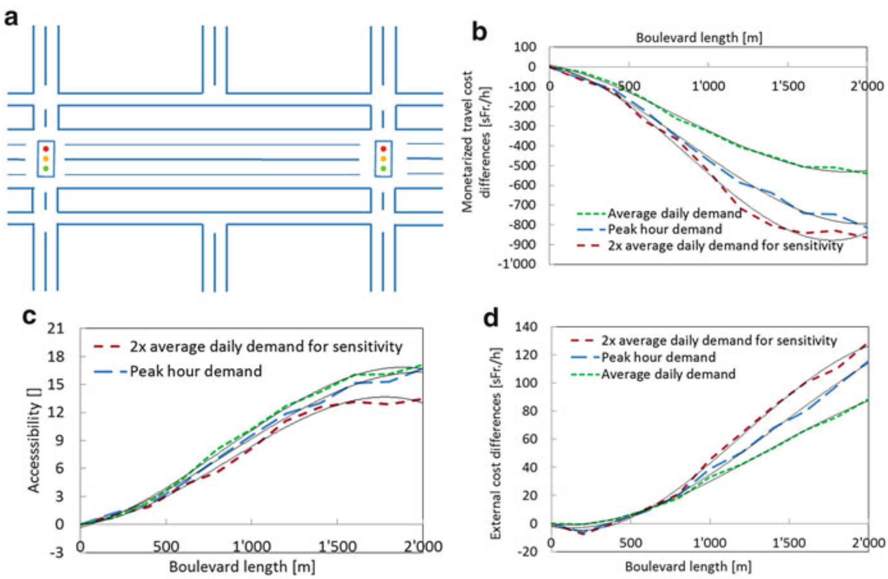

Fig. 7 Boulevard design, and the evaluation outcome as total monetarized travel time savings, accessibility changes and external effects for different boulevard lengths and network size $2 \cdot 2$ [km^2], including polynomial approximation (degree = 3). (**a**) Design of the boulevard integrated in a gridiron network. (**b**) Monetarized travel costs. (**c**) Accessibility. (**d**) External costs

a polynomial function $f(l)$. Polynomials of degree 3 are able to account for slope changes at both ends, to avoid overestimation of the data points, and still achieve a high fit (R^2). The highest slope values $f''(l_{\max}) = 0$ are determined, based on polynomials of s-shapes like in Fig. 7. Figure 7 shows delimited linear intervals, allowing elasticity calculation based on linear assumption. A two-sided application range $\{l_{\min}, l_{\max}\}$ is proposed, in which values close to the highest slope values can be expected. An approximated elasticity ϵ^s is calculated for the subset s of the data, which is inside the interval $l_{\min}$, $l_{\max}$. External costs have the highest elasticities (1.7–1.9), followed by accessibility (1.3–1.4), and travel costs (1.3–1.4).

Multiple issues occurred during modeling and evaluation, which are valuable for future design recommendations and the improvement of shape grammar rules. This additional information is added in the form of shape grammar rules including application specifications, and listed below. Vitins and Axhausen [48] add additional details regarding these rules.

Rule 1: Boulevards should have a certain minimum length to achieve noticeable travel cost reduction and increasing accessibility: The minimum length is 2–3 blocks. Longer boulevards reduce travel costs and increase accessibilities.

Rule 2: Boulevards with signals at the center road have the highest travel cost and accessibility elasticities $\epsilon^{s,T}$, $\epsilon^{s,A}$.

Rule 3: Travel speed on the boulevards has to be higher than on parallel roads, even when implementing signals with uniform waiting time.

Rule 4: The capacity of the center road has to be high enough to serve the flows. At least two center lanes for each directions are advisable.

Rule 5: Boulevards reduce generalized travel costs of urban traffic if the major intersections at the center road provide enough capacity and low turn delays for the required flows. This holds also for the crossings of the boulevard, which might be bottlenecks for crossing traffic. At least 3 approaching lanes for the major intersections are advisable.

Rule 6: Boulevards have a benefit-to-cost ratio of $\frac{b}{c} > 1.0$ only if the land prices are relatively low for acquisition. Obviously, boulevards as proposed above should be designed in an early stage of urban design. If not, additional economical studies and effects [e.g. 46] might be considered, beyond a standard cost–benefit procedure.

The above rules were derived with the following assumptions and evaluation methods (application specifications).

- A static model is used for all evaluations with detailed intersection delay calculations [HCM, 42], but ignoring spill-over effects.
- Urban density of 15,068 [pers/km^2] and 6,685 [jobs/km^2] are assumed, including peak hour and sensitivity flows which correspond to a 4-story perimeter development. Travel behavior is based on the Swiss census and travel diary [41].
- The boulevard is modeled in a gridiron network of $2 \cdot 2$[km^2], and $3 \cdot 3$[km^2], respectively.

4.3 Optimized Networks (Study Design 2)

Study design 2 requires optimal networks to extract shape grammar rules. Therefore, optimal networks are defined first which conform to the planning needs, such as efficiency, minimized infrastructure costs, safety, or other attributes. Two approaches are suggested to obtain these networks: First, it is possible to define real-world networks, which conform to the planning objectives. For example, Cardillo et al. [11] extracted network characteristics from multiple real-world networks. Second, artificial network models can be generated and optimized regarding the planning objectives. Detailed available models and underlying methods are available for transport simulation, some of them covering also urban simulations. Independent of these two approaches, it is required to have a large enough set of networks for statistical evaluations.

The second approach is applied in the following, and therefore, artificially designed networks are required for evaluations. For this purpose, a network design algorithm is applied, as described in [49, 50]. The algorithm deals with the nonlinear road assignment and complexity of the road network graph and therefore implements an integrated genetic and ant colony algorithm. The algorithm designs road networks or a limited spatial area, connecting demand generating blocks. Assumptions are required for the design process on the demand side, especially urban densities, and on the supply side, in particular infrastructure expenditures.

Study design 2 enables multiple evaluations, e.g. on topology or network element choice and densities. Exemplarily, the focus is on network meshedness in this section. The meshedness coefficient M (also in [10, 13]) is a sensitive graph topology measure which is defined in the following: $M = F/F_{\max} = F/(2N - 5)$, where F is the number of faces of a network graph, and $F_{\max}$ the maximum possible number of faces in a maximally connected planar graph ($F_{\max} = 2N - 5$), proportional to the number of nodes N [11, p. 5]. Compared to network element densities (nodes, links), and node degree, M focusses on the topology of networks by accounting for the face densities. Regarding the meshedness coefficient M, Cardillo et al. [11] found major differences on real-world networks. For example, networks of New York, Savannah, and San Francisco have values of $M > 0.3$, and in contrast to Irvine and Walnut Creek with $M < 0.1$, as shown in Table 1.

The upper half of Table 1 shows values of M for multiple network patterns. M differs considerably between the different patterns. The lower half of Table 1 shows values of M for optimized networks. These networks are modeled similar to the networks described in Sect. 4.2; however, the featureless plane is smaller ($1 \cdot 1$ [km^2]) due to the complex optimization. These networks were designed with different urban densities, travel demand, and infrastructure costs and optimized regarding low generalized user costs. Despite different intersection types, high values of M are achieved through the optimization process, including low standard deviation σ. There is evidence that networks with high M values correlate with low user costs (see also Sect. 4.4).

Table 1 Example networks (1·1 [mile^2]) and their meshedness coefficient M, based on [11], the figures in [24], and evaluations of the optimized networks

Design	Examples	M	σ	Example network
Real-world networks:				
Gridiron	Barcelona, Los Angeles, New York, Richmond, Savannah, San Francisco	0.291	0.0435	 Manhattan
Medieval	Ahmedabad, Cairo, Bologna, London, Venice, Vienna	0.229	0.0374	 Venice
Baroque	New Delhi, Washington	0.224	0.0695	 Washington
Modernist	Brasilia, Irvine (1)	0.116	0.0310	 Brasilia
Dentritic	Irvine (2), Walnut Creek	0.049	0.0350	 Irvine
Optimized networks:				
Ignoring turn delays		0.259	0.0450	
With signals		0.265	0.0411	
Right-of-way		0.266	0.0728	
With roundabouts		0.264	0.0258	

4.4 Rule Implementation (Study Design 3)

Exemplarily, this section focusses on a straightforward application of study design 3 on intersection type choice. Different intersection types are separately applied in the design of networks already mentioned in Sect. 4.3. One intersection type is applied in a single network, and entire networks with different implemented intersection types are designed similar to Sect. 4.3, and compared with each other (Fig. 5d).

Beside the rather straightforward intersection type evaluations, further examples can be found, e.g., for road type choice in network design [50].

Signals, right-of-way intersections, and roundabouts are considered in the following. All-way stop controlled intersections are ignored due to their general absence outside the USA. All intersection types are modeled with two approaching lanes for comparison reasons. Conflicting flows increase turn delays, and are considered in the delay calculations, which are based on the HCM [42].

Intersection types are sensitive to flow changes. On the supply side, variable infrastructure expenses lead to various network densities which will be considered in the following. In addition, the maximum supply approach is applied (Sect. 3.6), which means that structural densities (jobs, population) are increased until the additional travel time has reached 20 % of the free flow travel times $\left(\Delta d_{\text{pop,jobs}}\left(\Delta t^{20\,\%}\right)\right)$. Table 2 shows the relevant independent variable after a stepwise regression, when considering $\Delta d_{\text{pop,jobs}}\left(\Delta t^{20\,\%}\right)$ as dependent variable. Additionally, 2 scenarios are defined for road and intersection type sensitivity. Scenario 1 consists of street types with doubled capacity compared to scenario 0, scenario 2 consists of intersection types with an additional approaching lane; calculations are based on the HCM [42].

As shown in Table 2, d_r and M contribute significantly to high supply, in scenario 0 and 1. Again, low multicollinearity is expected between d_r and M due to higher VIF values. Additionally, the implementation of signals is significant. Right-of-way intersections only have a positive significant influence in scenario 1, but still lower than signals. The positive influence of right-of-way intersections mirrors that roundabouts might have higher delays at increasing densities. In scenario 2, right-of-way intersections have negative influence, indicating that capacities of right-of-way

Table 2 Regression result of optimized networks for $\Delta d_{\text{pop,jobs}}\left(\Delta t^{20\,\%}\right)$ as a dependent variable

Scenario	Parameter	Significance	Stand. β	Variance inflation factor (VIF)
0	Meshedness M	0.009	0.286	1.310
	Network density d_r	0.000	0.409	1.357
	Dummy signal	0.000	0.665	1.060
Significance: 0.000				
Adj. R^2: 0.688				
1	Meshedness M	0.011	0.273	1.310
	Network density d_r	0.000	0.434	1.362
	Dummy right-of-way	0.000	0.493	1.197
	Dummy signal	0.000	0.707	1.266
Significance: 0.000				
Adj. R^2: 0.705				
2	Meshedness M	0.000	0.621	1.002
	Dummy right-of-way	0.011	−0.307	1.193
	Dummy signal	0.043	0.239	1.193
Significance: 0.000				
Adj. R^2: 0.586				

intersections cannot increase to the same extent than at roundabouts and signals. Right-of-way intersections are therefore less efficient with additional lanes, and increasing flows, which also comports with the following results on intersection type choice.

In Table 2, scenario 2 shows significant values for M, signals, and right-of-way intersections. Additionally, d_r is significant even after the stepwise linear regression. However, the standardized β value of d_r is negative, in combination with M, which seems unreasonable. M and d_r are slightly correlating, as shown in scenario 0 and 1. Therefore, d_r is excluded in Table 2 in scenario 2 due to low standardized β and lower significance, compared to M.

It is therefore concluded that signals produce lower turn delays at high flows. At lower flows, right-of-way intersections have low turn delays. No significant correlation could be found for 3 and 4 arm intersections, even though they were considered in the evaluation. However, a detailed evaluation of signal delays at an isolated intersection shows lower delays at 4 arms, due to a more optimized phase allocation.

5 Conclusion

It is necessary to consolidate fundamental knowledge of shape grammars for future advances in urban planning, due to their advantages in application, and the already increasing number of shape grammar implementations. For this purpose, evaluations are required to measure the effect of rules. This chapter shows that it is possible to evaluate the effect of certain shape grammars in planning, based on well-defined, but exchangeable objective functions. The proposed maximum supply approach applied in shape grammar rule evaluation accounts for variable urban densities and reliability. Knowledge about the application perimeter and limitations of the grammar rules is fundamental for future and meaningful applications. Both aspects, the evaluation and the limitations of the rules, comports with theory and the general concept of the language approach. In the future, evaluations of additional rules will be necessary. Moreover, evaluations on urban design and architectural rules are required to increase the understanding in cognate fields.

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